The Kingpin Axis

How Forces and Moments at the Contact Patch or Wheel Center are Converted to a Steering Rack Force

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About the author

Tim Drotar is currently a senior staff engineer in vehicle dynamics at Tesla. Prior to joining Tesla, he spent 30 years at Ford Motor Company where he specialized in chassis systems and vehicle dynamics for passenger cars and light trucks. Tim is a member of SAE, SCCA and The Tire Society. He holds a B.S. in Mechanical Engineering from Lawrence Technological University and a M.S. in Mechanical Engineering from the University of Michigan-Dearborn.

Tim also teaches the following classes for SAE:

- Advanced Vehicle Dynamics for Passenger Cars and Light Trucks
  - https://www.sae.org/learn/content/c0415/
- Fundamentals of Steering Systems
  - https://www.sae.org/learn/content/c0716/

Outline

- Learning Objectives
- References
- Defining the kingpin axis
- Parameters associated with the kingpin axis
- Components that make up the kingpin moment
- Axle kingpin moment
- The steer arm
- The kingpin axis in the Systems Engineering V
- Summary

Learning Objectives

At the end of this presentation, you should be able to:

- Identify the kingpin axis for suspension types commonly found on passenger cars and trucks
- Define key parameters related to the location of the kingpin axis relative to the wheel center and contact patch
- Calculate the total moment about the kingpin axis due to forces and moment at the wheel center and contact patch
- Differentiate between front and rear steer
- Identify the steer arm
- Calculate the vector sum force at the steer arm required to counteract the kingpin moment
- Calculate the axial rack force required to counteract the kingpin moment

References

These notes were created from several sources, including:

Gillespie, T. Fundamentals of Vehicle Dynamics. Warrendale, PA: Society of Automotive Engineers, 1992.

Reimpell, J., Stoll, H., Betzler, J. The Automotive Chassis: Engineering Principles. Woburn, MA: Butterworth-Heinemann, 2001

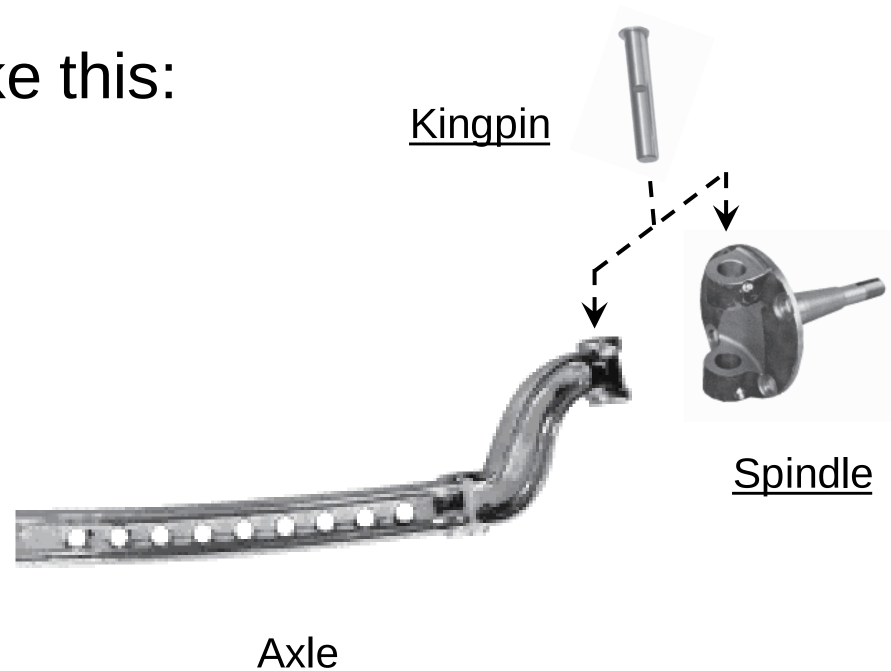
Harrer, M., Pfeffer, P. Steering Handbook. Switzerland: Springer International, 2017.

The Kingpin Axis

The **kingpin axis**, sometimes referred to as the steering axis, is the 3-dimensional elasto-kinematic axis about which the wheel rotates.

In old days, a kingpin looked like this:

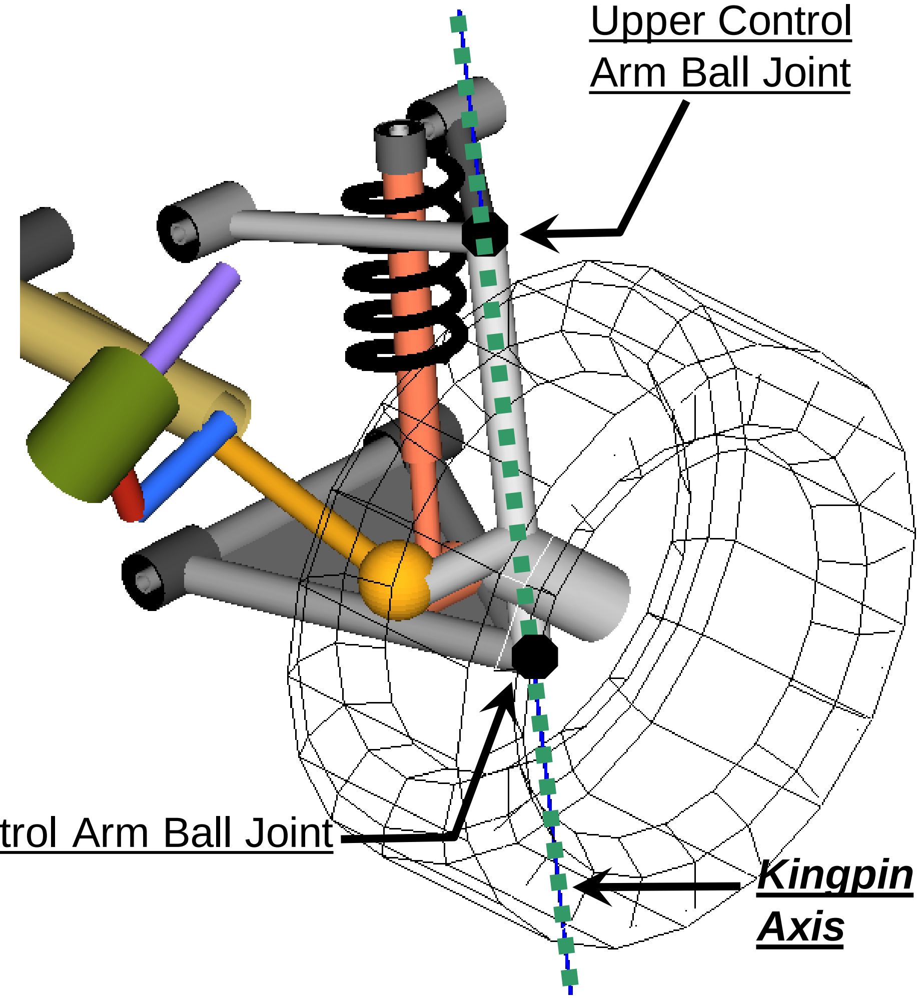
www.hfmgv.org/exhibits/showroom/1896d/duryea.html

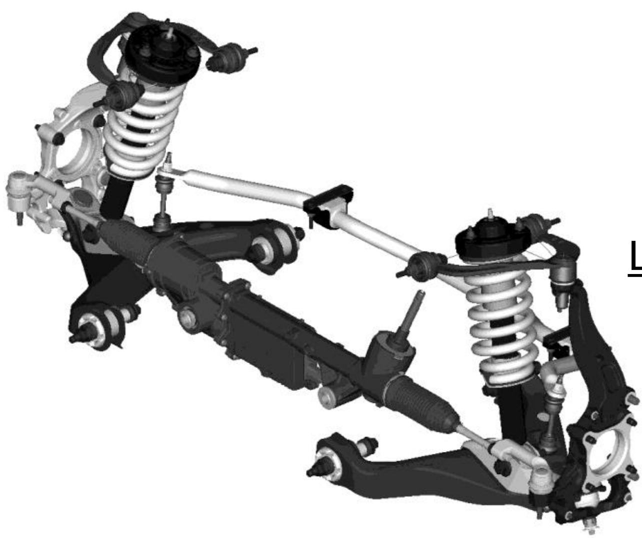


http://www.peteandjakes.com/superbell/index.html

For a modern suspension, the kingpin axis is a “virtual axis”.

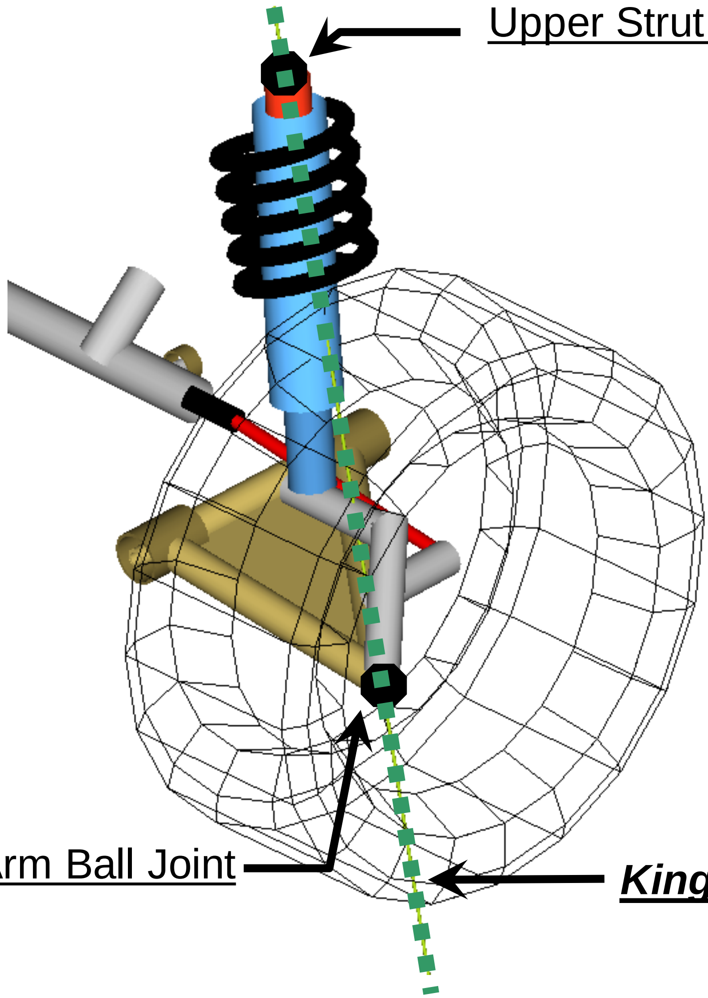
For a <u>SLA front suspension</u>, the kingpin axis is found by drawing a line between the center of the upper control arm ball joint and the lower control arm ball joint





The Kingpin Axis

For a <u>McPherson Strut</u> front suspension, the kingpin axis is found by drawing a line through the upper strut pivot point and the center of the lower control arm ball joint

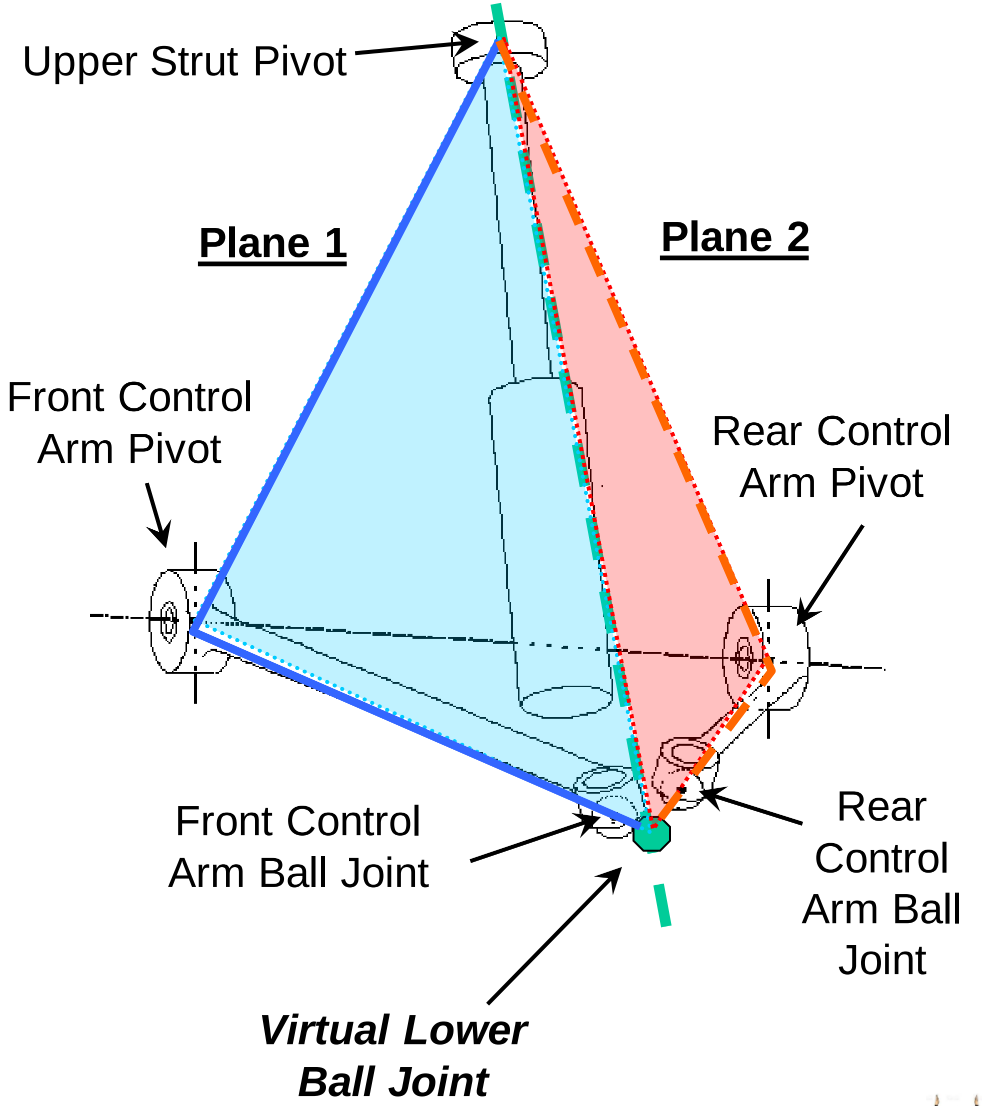


The Kingpin Axis

For a McPherson strut with double ball joint lower arm, the kingpin axis is found by the intersection of the following planes:

Plane 1:
- Upper strut pivot
- Front control arm pivot
- Front control arm ball joint

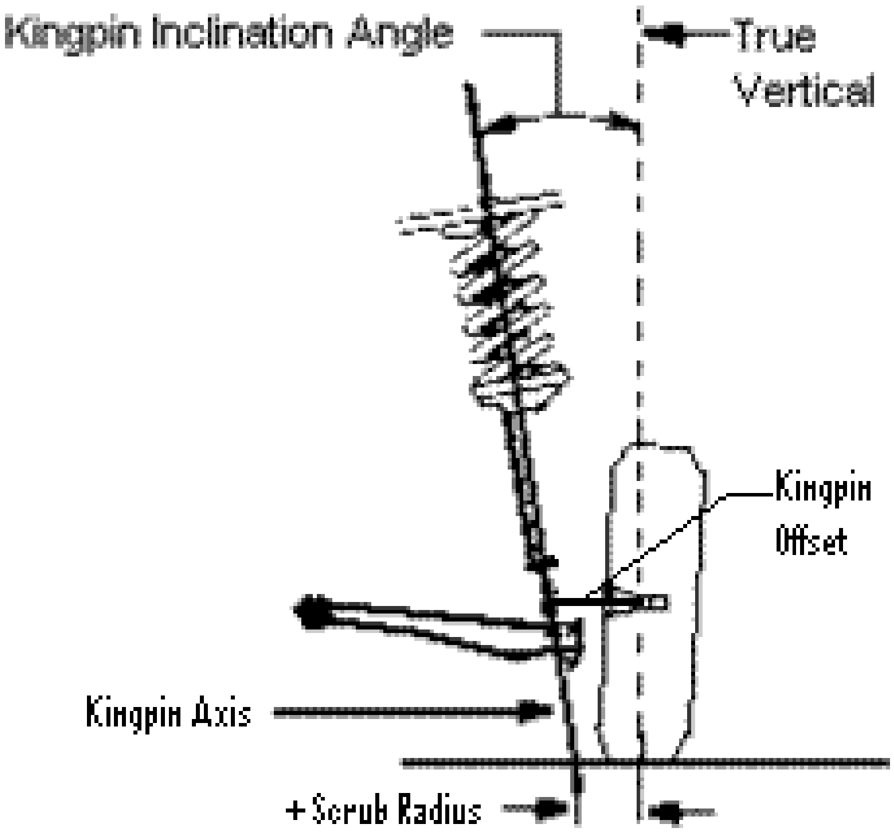
Plane 2:
- Upper strut pivot
- Rear control arm pivot
- Rear control arm ball joint



Parameters Associated with the Kingpin Axis

Looking at the kingpin axis in the rear view, we define some key parameters…

r_s = Scrub Radius
r_{kp} = Kingpin Offset
λ = Kingpin Inclination Angle

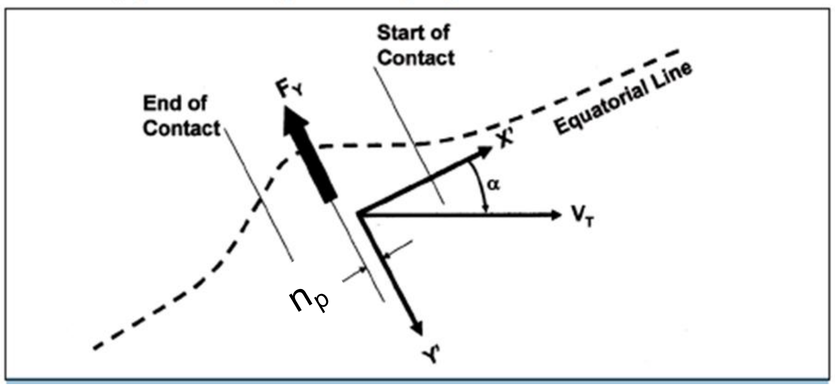


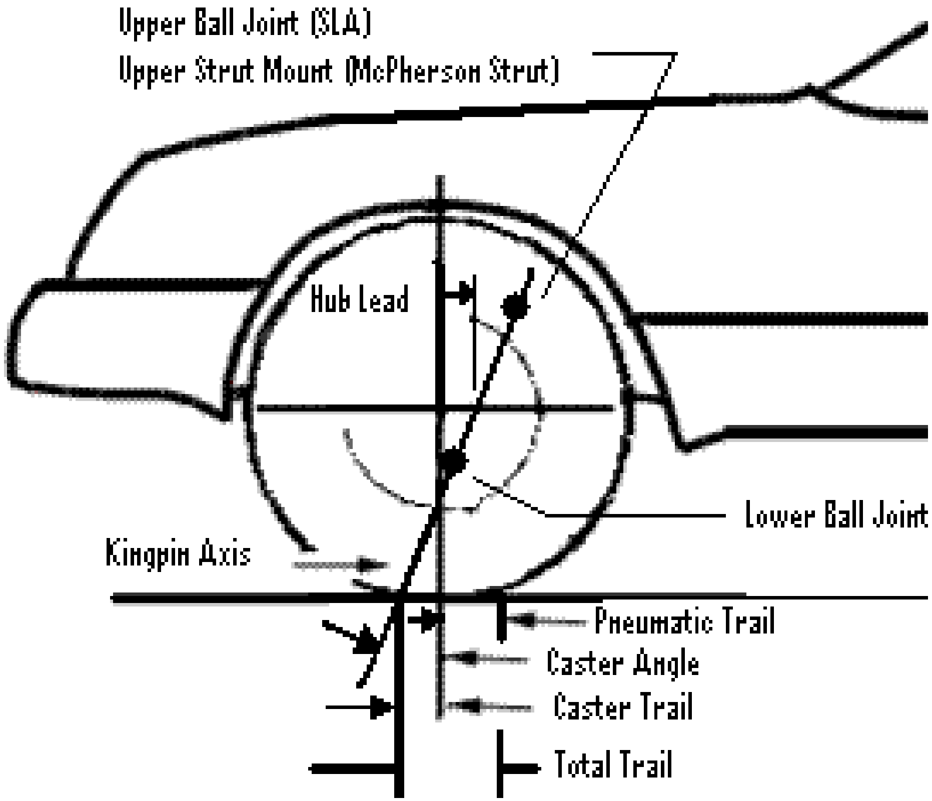
Parameters Associated with the Kingpin Axis

Looking at the kingpin axis in the side view

n_T = total trail
n_p = tire pneumatic trail
n_m = caster (mechanical) trail
n_{kp} = hub lead
ν = caster angle

Figure 8.3: String tire at a slip angle in contact with the road

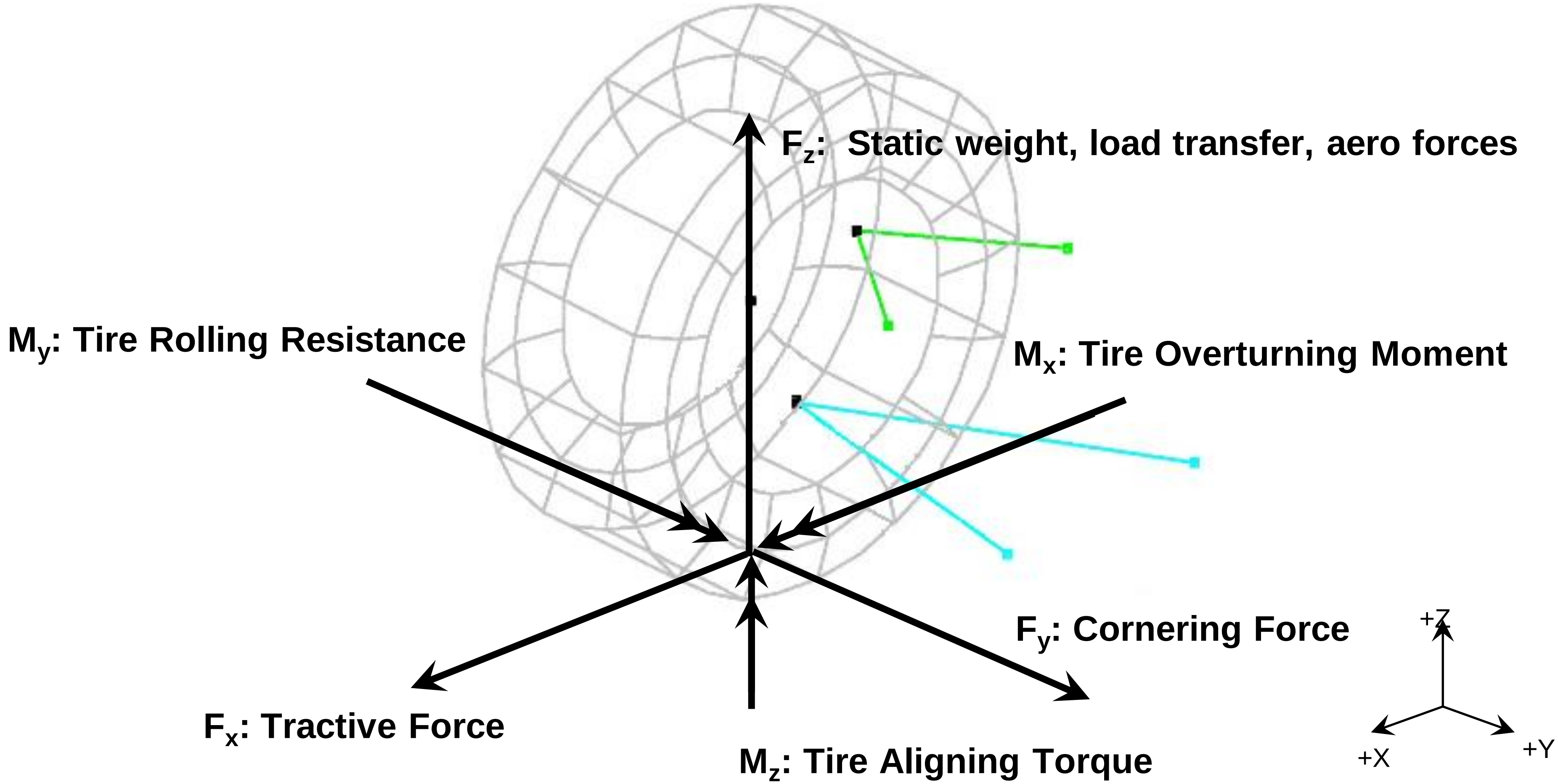




<u>Total Trail = Caster Trail + Pneumatic Trail</u>

Forces and Moments at the Contact Patch

Driving down the road, there are various forces and moments being generated at the contact patch:



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The Kingpin Moment

The combination of forces and moments create a total moment about the kingpin axis

  Moment due to tractive force (F_x)

+ Moment due to cornering force (F_y)

+ Moment due to vertical force (F_z)

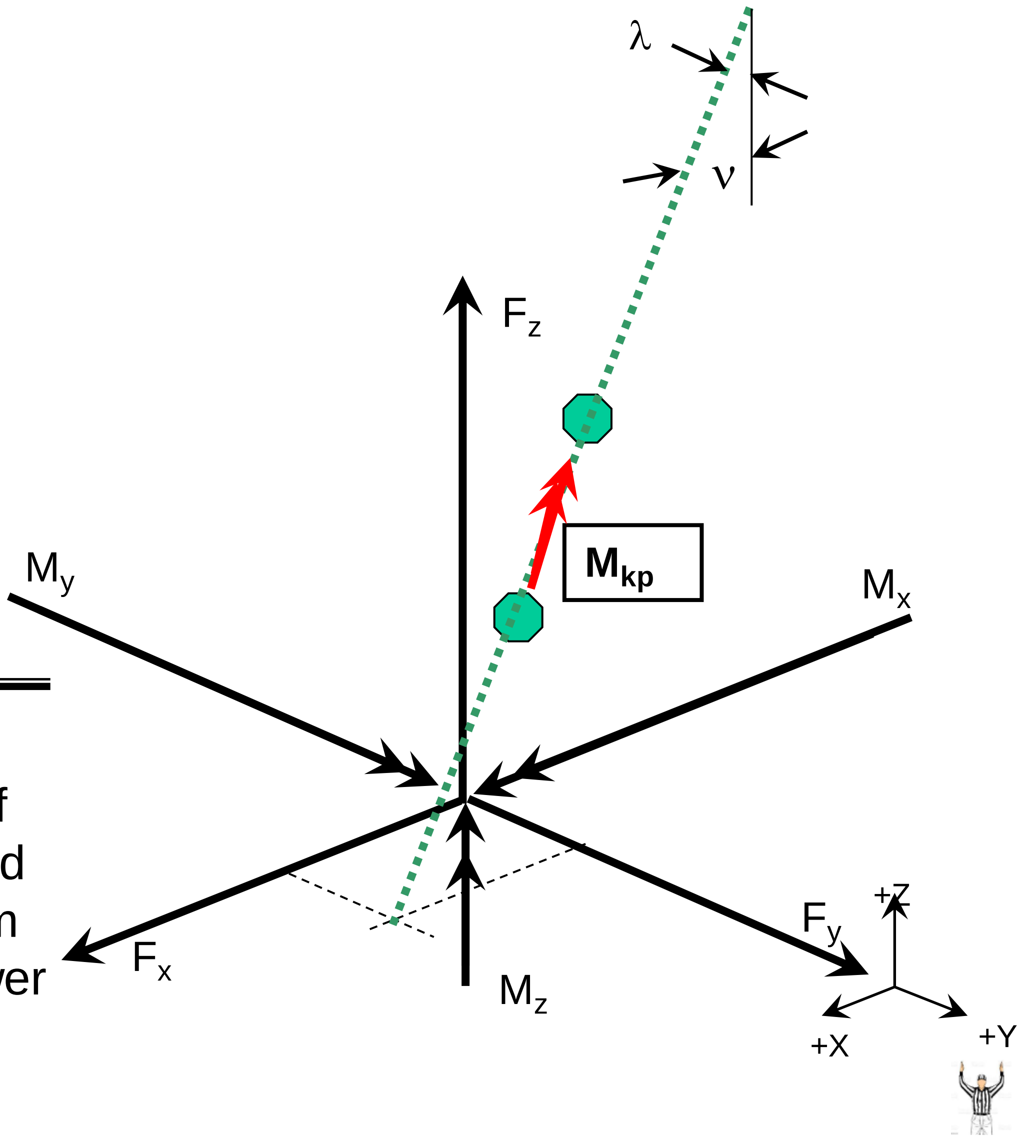
+ Tire overturning moment (M_x)

+ Tire rolling resistance moment (M_y)

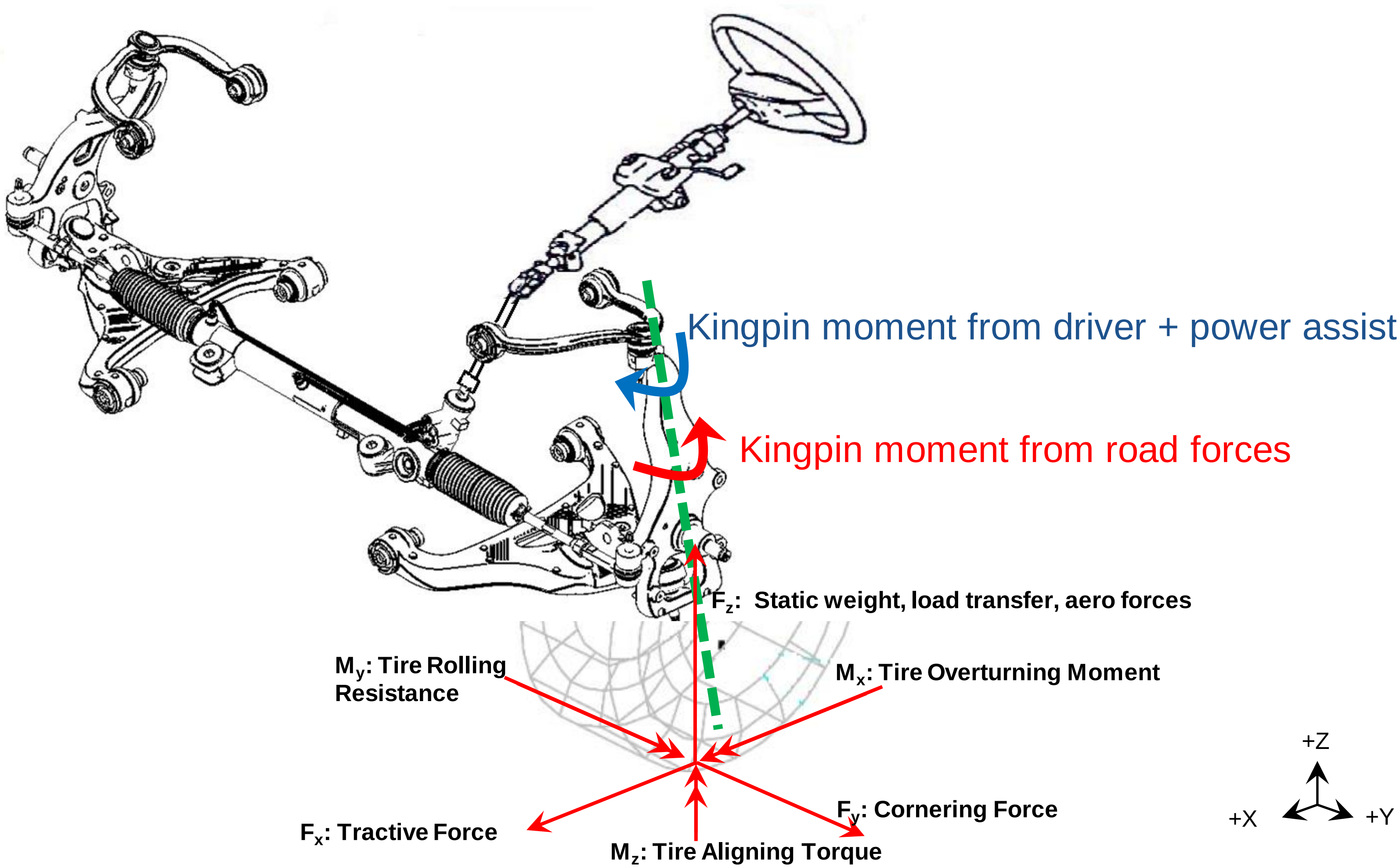
+ Tire aligning Moment (M_z)

= Total Moment About Kingpin Axis (M_{kp})

To keep the vehicle system in a state of equilibrium, this moment is counteracted by a combination of muscular force from the driver and assist force from the power steering system.



The steering system from the hand wheel to the roadwheel



The Kingpin Moment

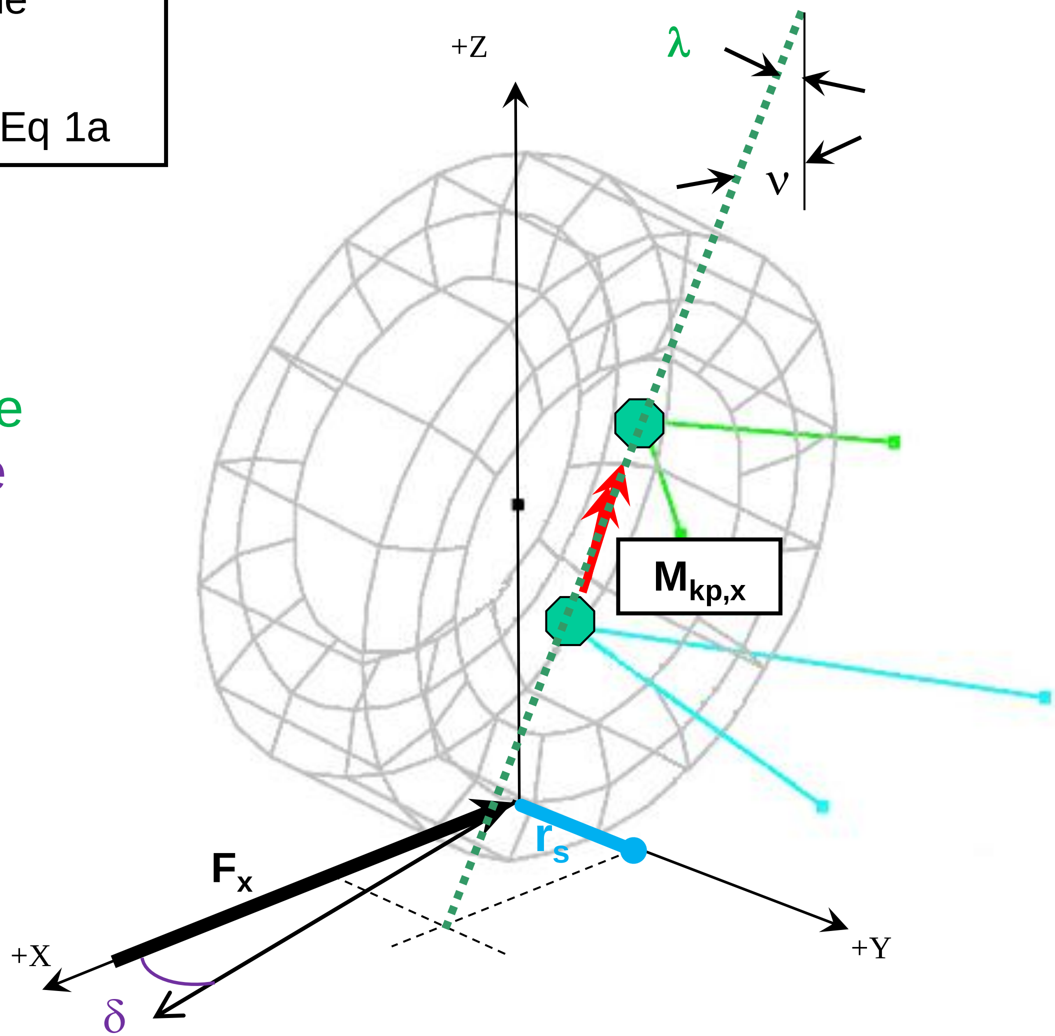
We’ll now the individual components that make up the total moment about the kingpin axis

Kingpin Moment due to Braking Force at CP

For the case of a braking force at the contact patch:

$$M_{kp,x} = -F_x * r_s * \cos(\lambda) * \cos(\delta) \quad .... \text{ Eq 1a}$$

r_s = Scrub radius
λ = Kingpin inclination angle
δ = Road wheel steer angle



Kingpin Moment due to Tractive Force at WC

For the case of a front wheel drive vehicle, the tractive force is applied at the wheel center:

$$M_{kp,dt} = F_{x,o} * r_{kp} * \cos(\lambda) * \cos(\delta) \quad \ldots\text{Eq 1b}$$

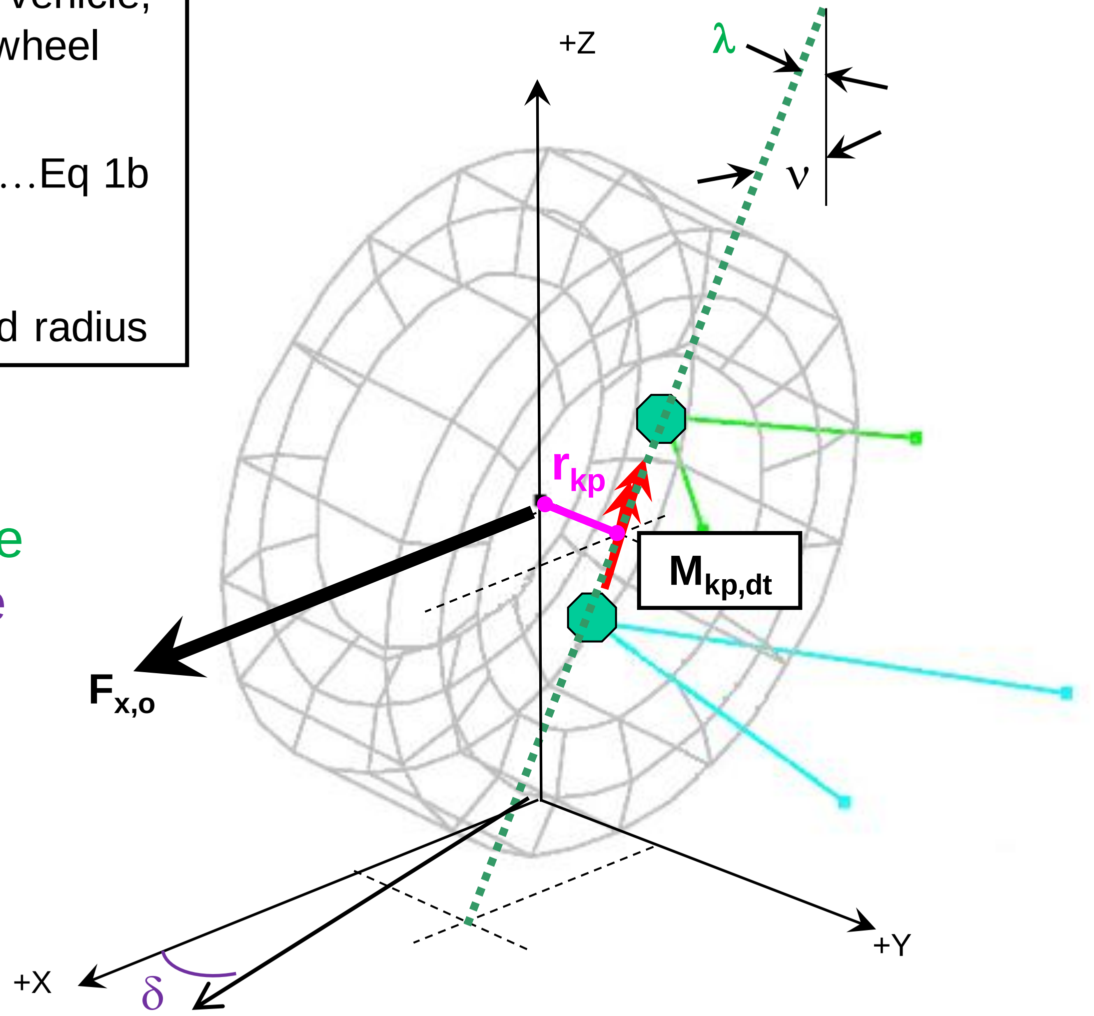
Where:

$F_{x,o}$ = drive torque / tire loaded radius

r_{kp} = Kingpin offset

λ = Kingpin inclination angle

δ = Road wheel steer angle

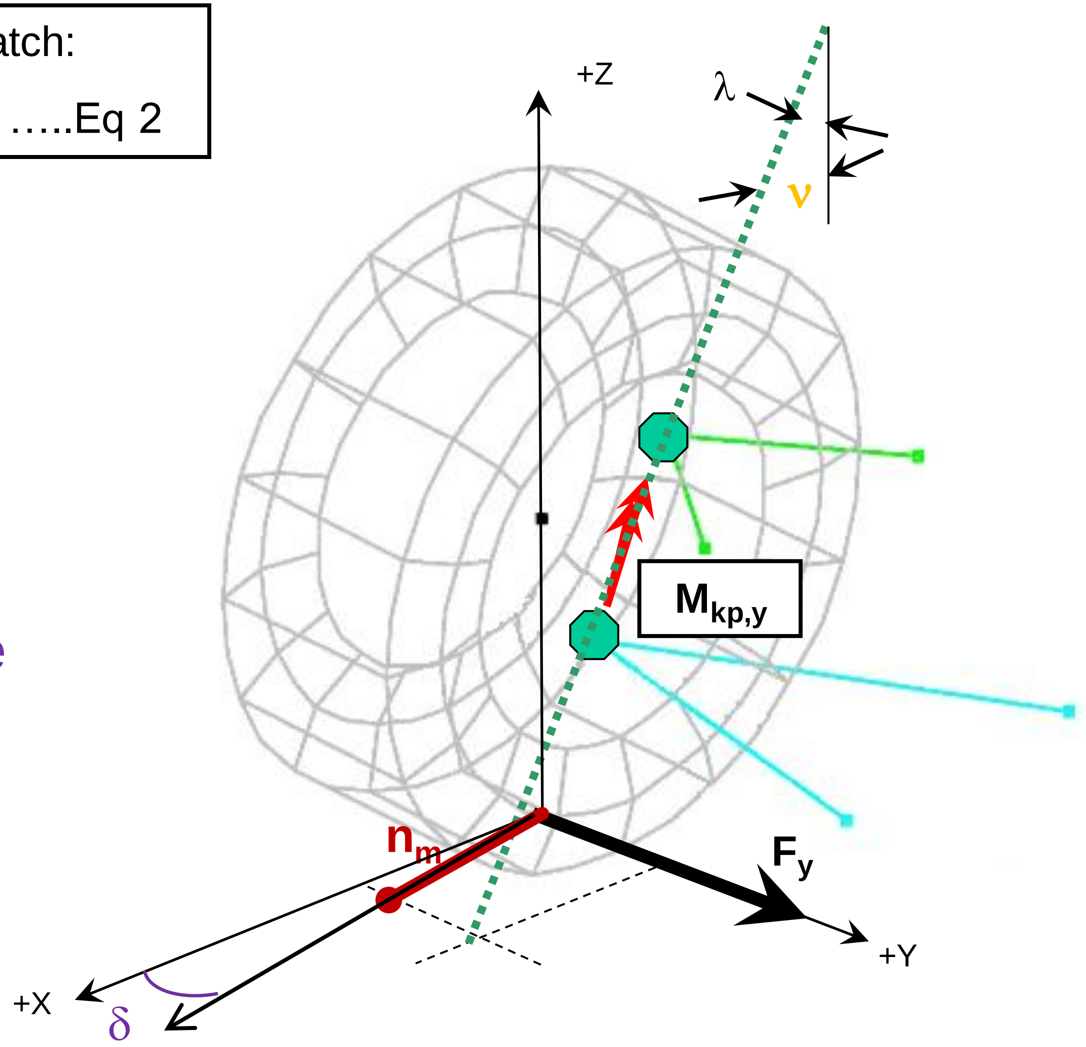


This equation can also be used in the case of regenerative braking, where a –Fx is applied to the WC

For a lateral force at the contact patch:

$M_{kp,y} = -F_y*\{\mathbf{n_m}*\cos(\nu)\}*\cos(\delta)$ …..Eq 2

n_m = Mechanical trail
ν = Caster angle
δ = Road wheel steer angle

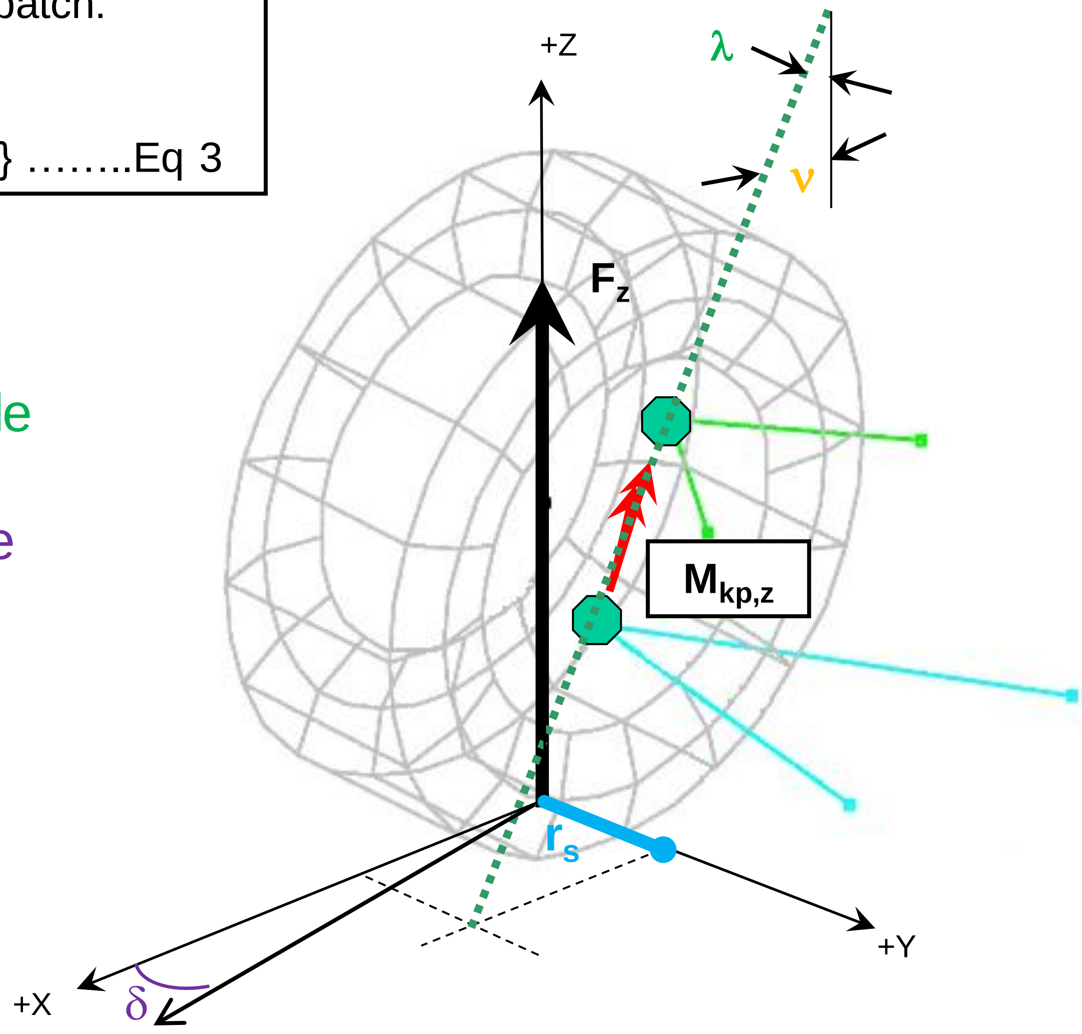


Kingpin Moment due to Vertical Force at CP

For a vertical force at the contact patch:

$$M_{kp,z} = -\{F_z*\sin(\lambda)\}*\{r_s*\sin(\delta)\}$$
$$+ \{F_z*\sin(\nu)\}*\{r_s*\cos(\delta)\} \text{ ....Eq 3}$$

r_s = Scrub radius
λ = Kingpin inclination angle
ν = Caster angle
δ = Road wheel steer angle



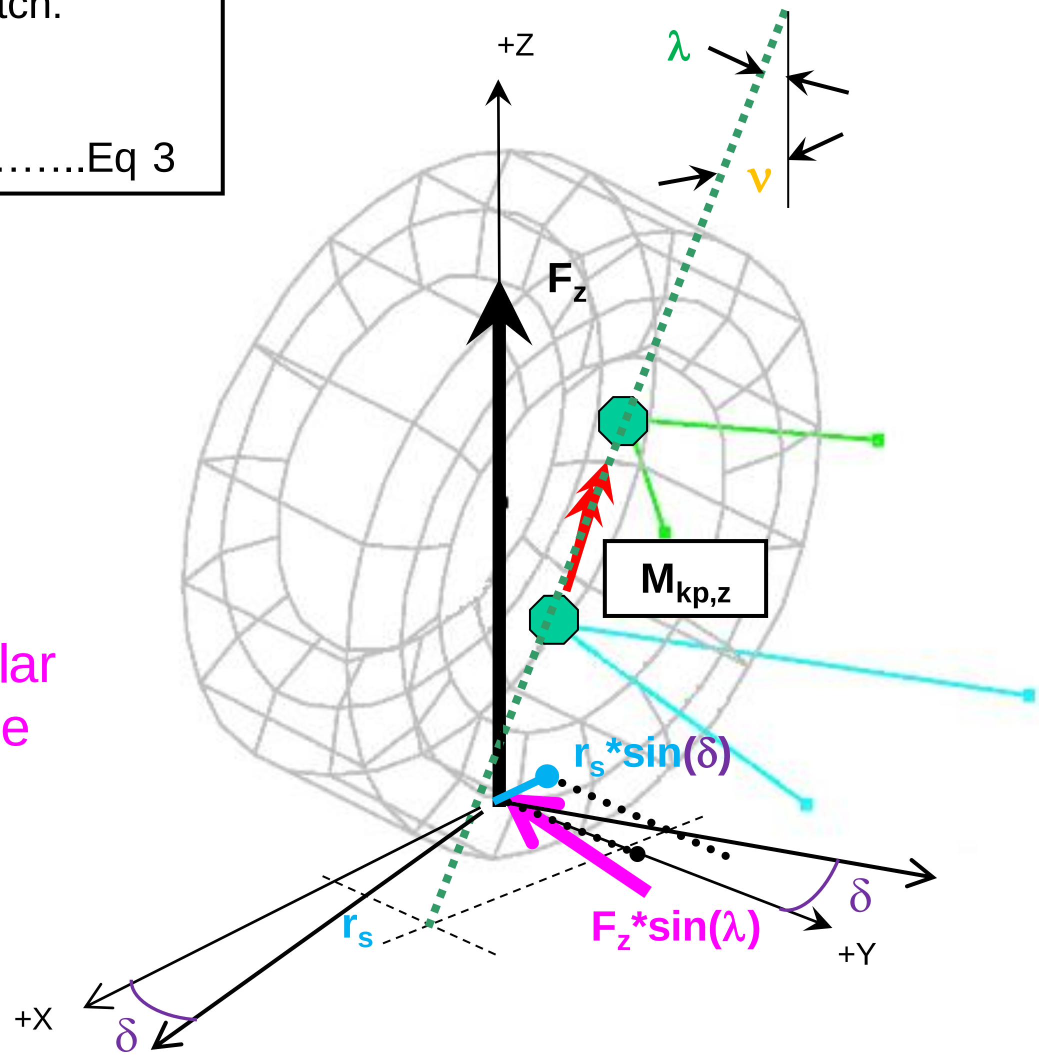
Kingpin Moment due to Vertical Force at CP – 1st Term

For a vertical force at the contact patch:

$$M_{kp,z} = -\{F_z{*}\sin(\lambda)\}{*}\{r_s{*}\sin(\delta)\}$$
$$+ \{F_z{*}\sin(\nu)\}{*}\{r_s{*}\cos(\delta)\} \text{ ....Eq 3}$$

r_s = Scrub radius
λ = Kingpin inclination angle
ν = Caster angle
δ = Road wheel steer angle

<u>$F_z{*}\sin(\nu)$</u>
Component of F_z, perpendicular to kingpin axis in the Y-Z plane



Kingpin Moment due to Vertical Force at CP – 2nd Term

For a vertical force at the contact patch:

$$M_{kp,z} = -\{F_z*\sin(\lambda)\}*\{r_s*\sin(\delta)\}$$

$$+ \{F_z*\sin(\nu)\}*\{r_s*\cos(\delta)\} \quad \text{........Eq 3}$$

r_s = Scrub radius

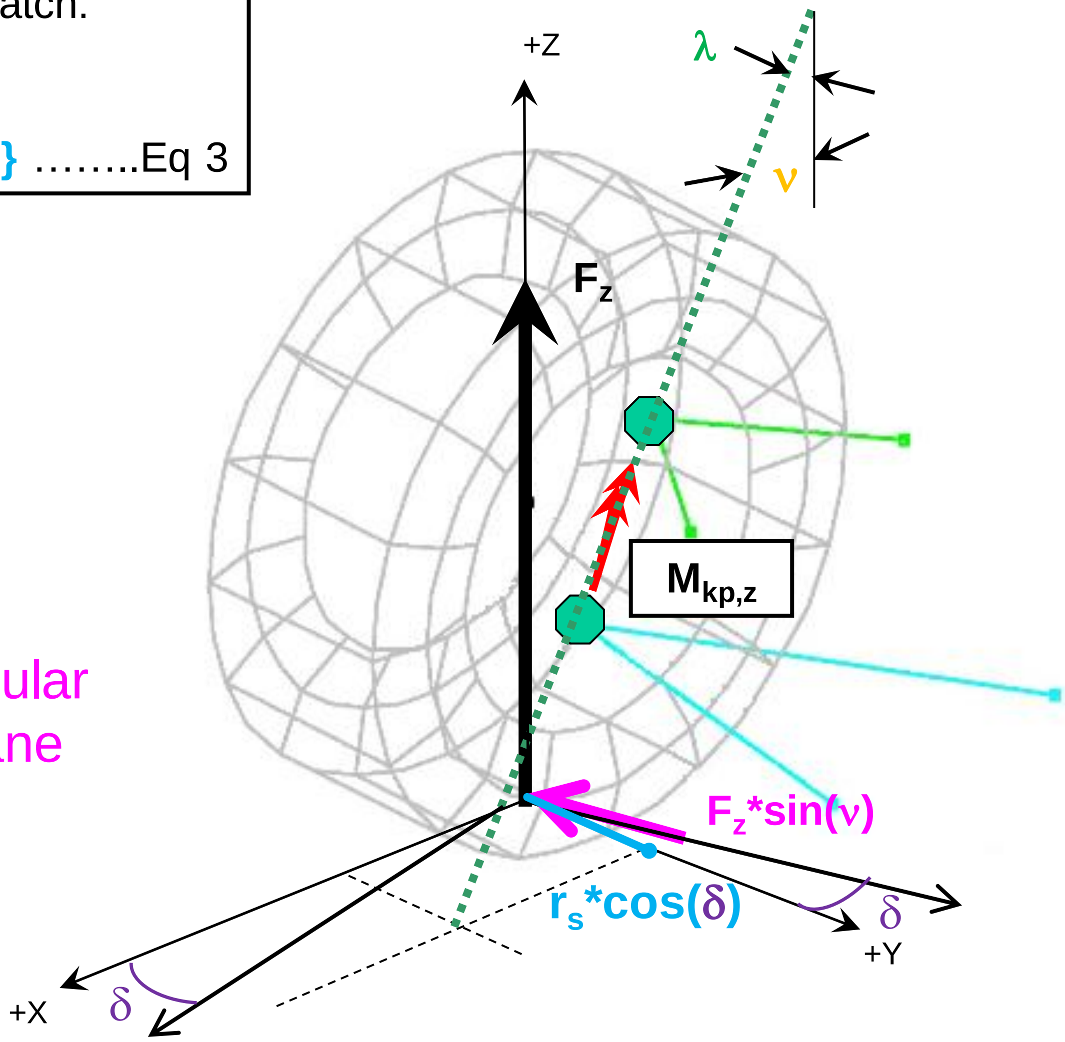
λ = Kingpin inclination angle

ν = Caster angle

δ = Road wheel steer angle

$F_z*\sin(\nu)$

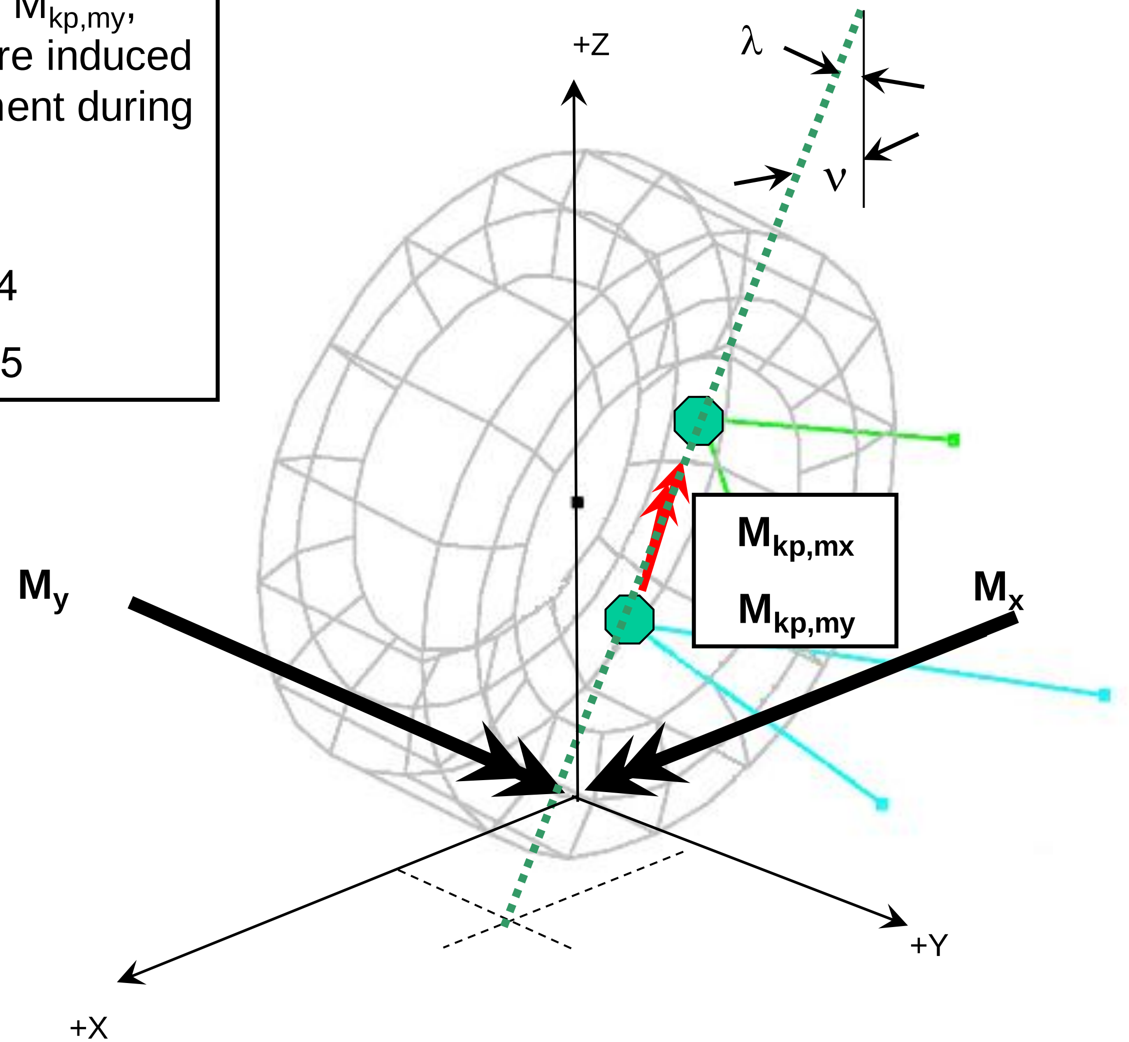
Component of F_z, perpendicular to kingpin axis in the X-Z plane



$M_{kp,mx}$, overturning moment, and $M_{kp,my}$, rolling resistance moment, are tire induced and create minimal kingpin moment during normal operating conditions

$$M_{kp,mx} \sim 0 \quad \text{........ Eq 4}$$

$$M_{kp,my} \sim 0 \quad \text{......... Eq 5}$$



Kingpin Moment due to Aligning Torque

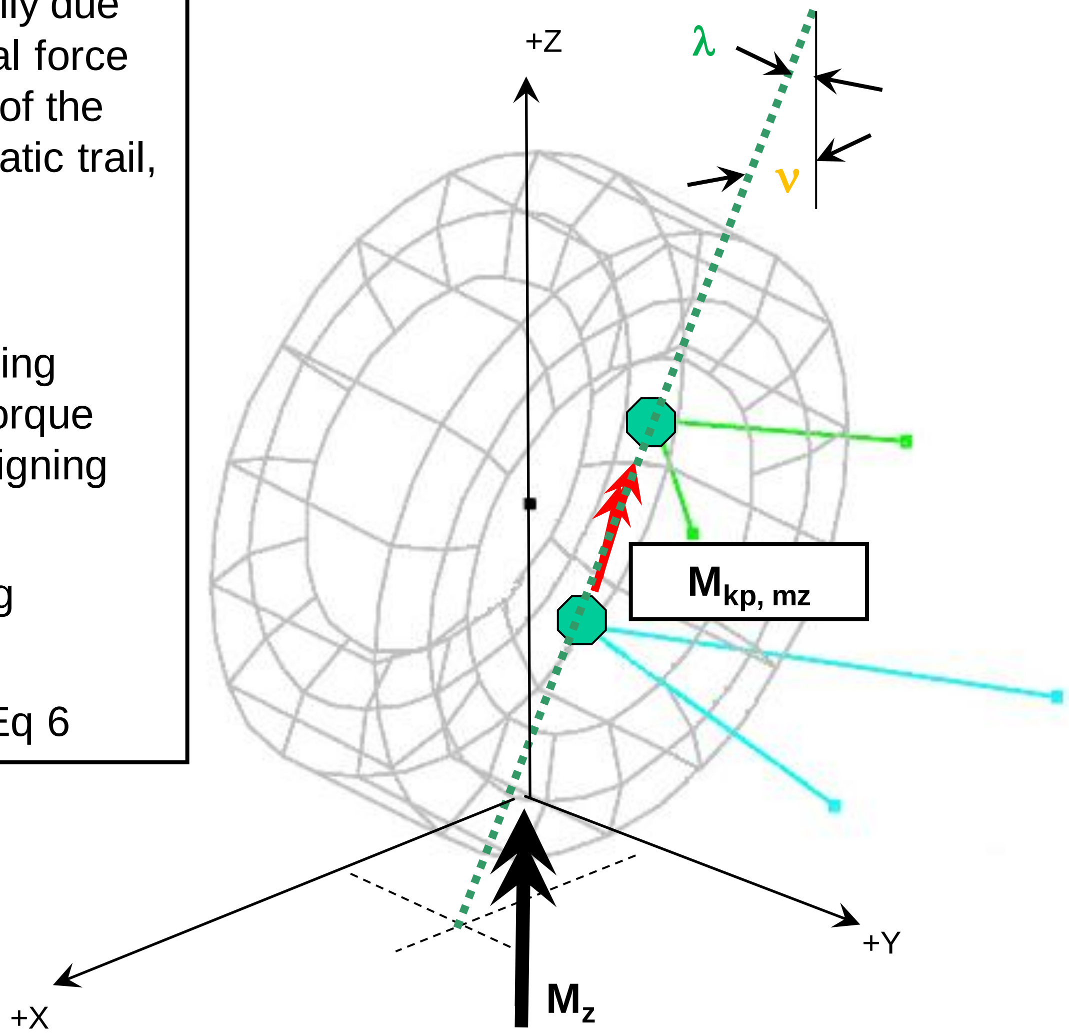
The aligning moment, M_z, is primarily due to the fact that the centroid of lateral force is offset from the geometric center of the tire by a distance called the pneumatic trail, n_p. Thus:

$$M_z = F_y * n_p$$

A smaller source of additional aligning moment is the Residual Aligning Torque (RAT), which is the non-zero tire aligning torque at zero lateral force.

The kingpin moment due to aligning moment is:

$$M_{kp,mz} = M_z * \cos\{(\lambda^2+\nu^2)^{0.5}\} \quad \ldots\text{Eq 6}$$



λ = Kingpin inclination angle

ν = Caster angle

Kingpin Moment from Steering a 'Locked' Tire

During a static parking event, where the steering wheel is turned while the brakes are applied, the moment generated about the kingpin axis is:

$$M_{kp,sp} = -\mu * F_z * [\{r_s * \cos(\lambda)\}^2 + \{n_T * \cos(\nu)\}^2]^{0.5}$$

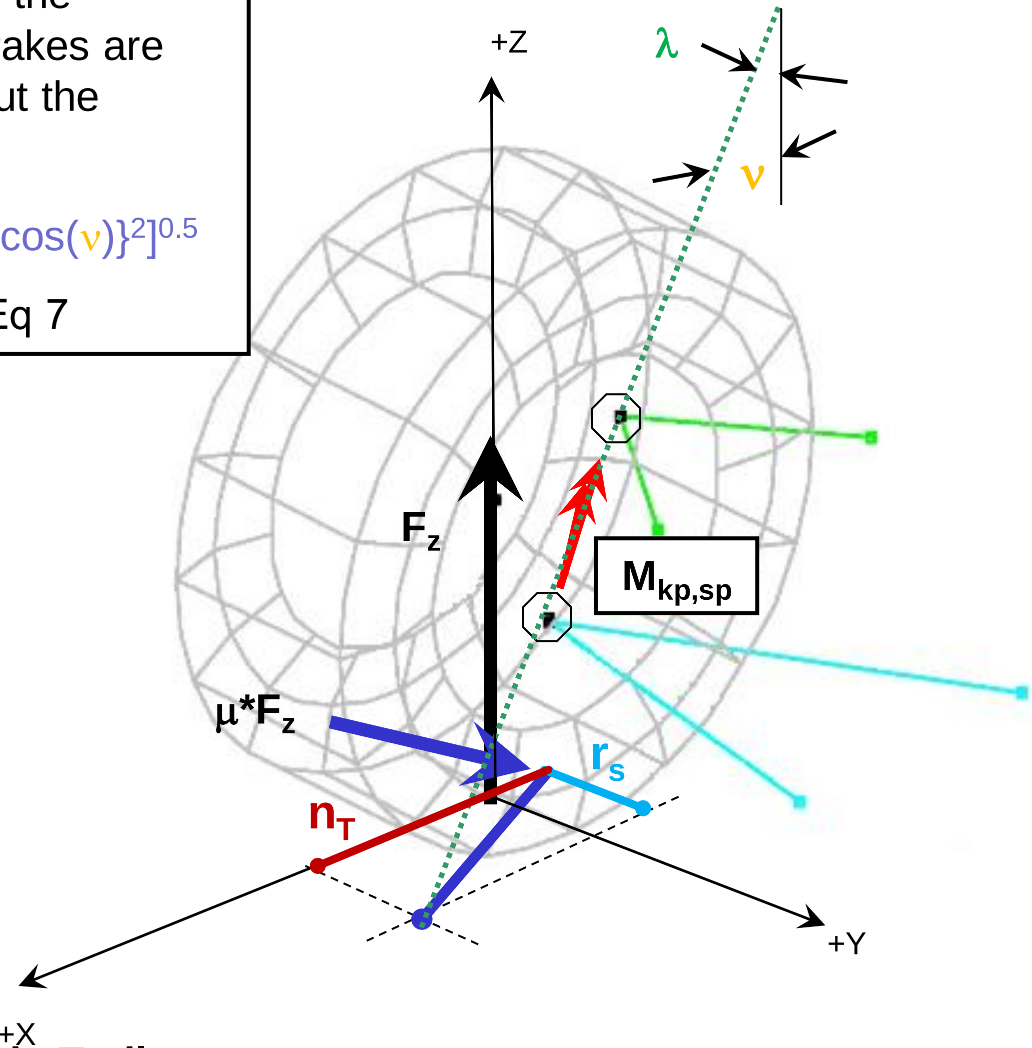
….. Eq 7

r_s = Scrub radius

λ = Kingpin inclination angle

n_T = Total trail

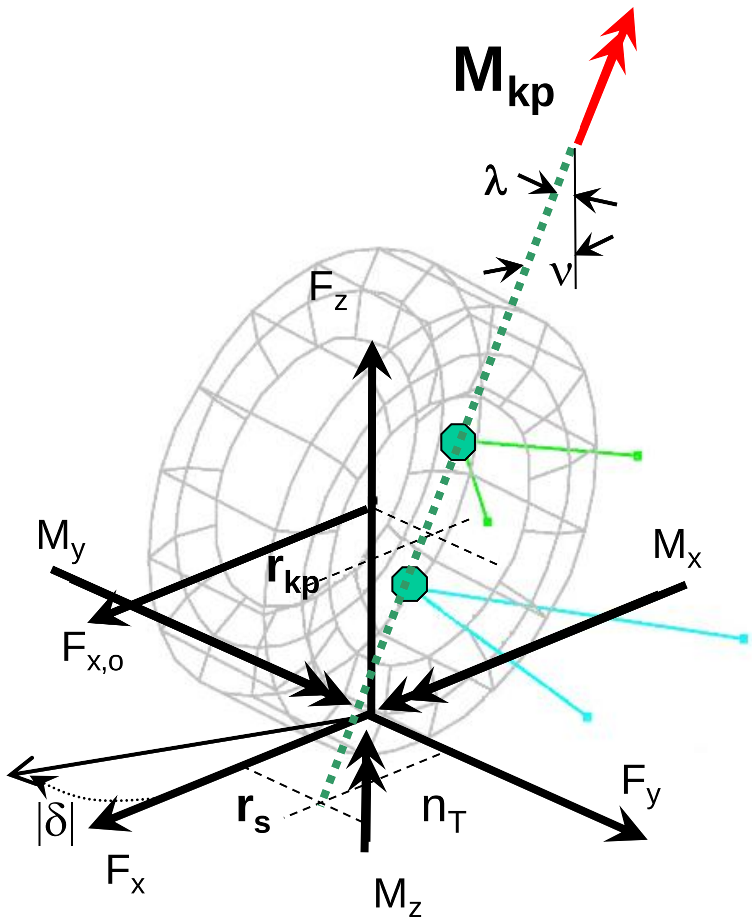
ν = Caster angle



Total Trail = Caster Trail + Pneumatic Trail

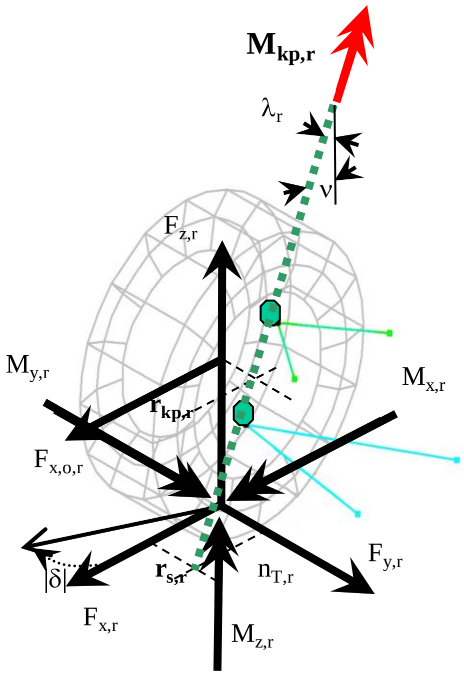
Summary of Equations

Braking Force at Contact Patch	$M_{kp,fx} = -F_x*r_s*\cos(\lambda)*\cos(\delta)$	Eq 1a
Longitudinal Force at Wheel Center	$M_{kp,dt} = F_{x,o}*r_{kp}*\cos(\lambda)*\cos(\delta)$	Eq 1b
Lateral Force at Contact Patch	$M_{kp,fy} = -F_y*\{n_m*\cos(\nu)\}*\cos(\delta)$	Eq 2
Vertical Force at Contact Patch	$M_{kp,z} = -\{F_z*\sin(\lambda)\}*\{r_s*\sin(\delta)\} + \{F_z*\sin(\nu)\}*\{r_s*\cos(\delta)\}$	Eq 3
Overturning Moment	$M_{kp,mx} \sim 0$	Eq 4
Rolling Resistance Moment	$M_{kp,my} \sim 0$	Eq 5
Aligning Moment	$M_{kp,mz} = M_z*\cos\{(\lambda^2+\nu^2)^{0.5}\}$	Eq 6
Static Park	$M_{kp,sp} = -\mu*F_z*[\{r_s*\cos(\lambda)\}^2 + \{n_m*\cos(\nu)\}^2]^{0.5}$	Eq 7

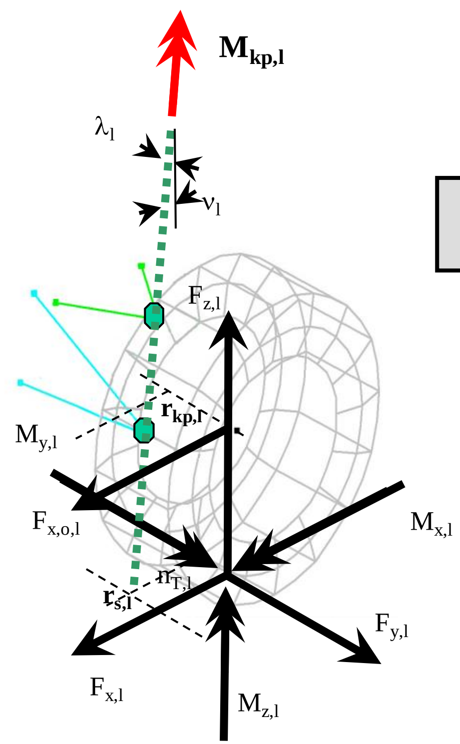


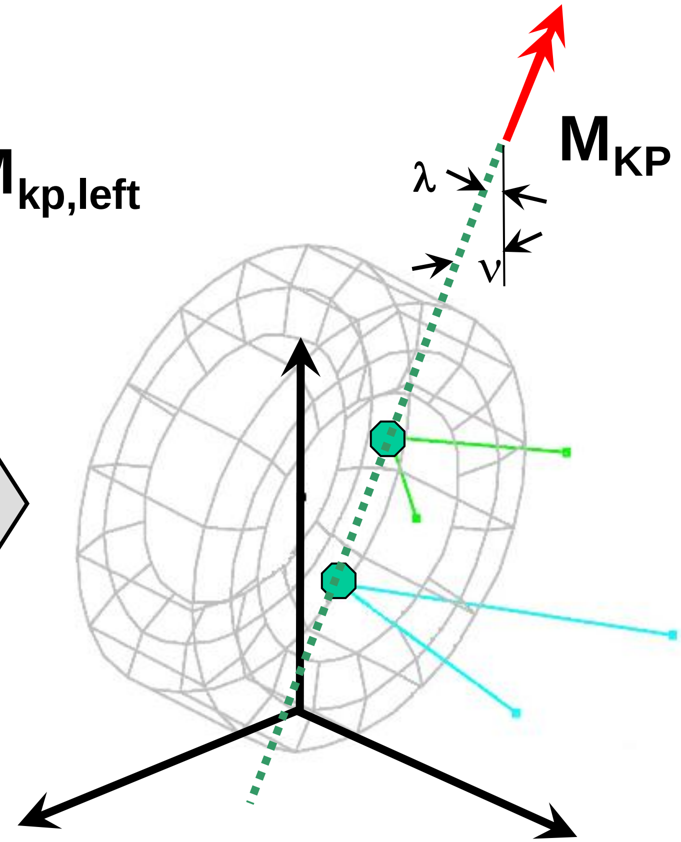
Axle Kingpin Moment

We can take a “bicycle model” approach to the analysis where the axle kingpin moment is the sum of the right and left kingpin moments:



$$M_{KP} = M_{kp,right} + M_{kp,left}$$





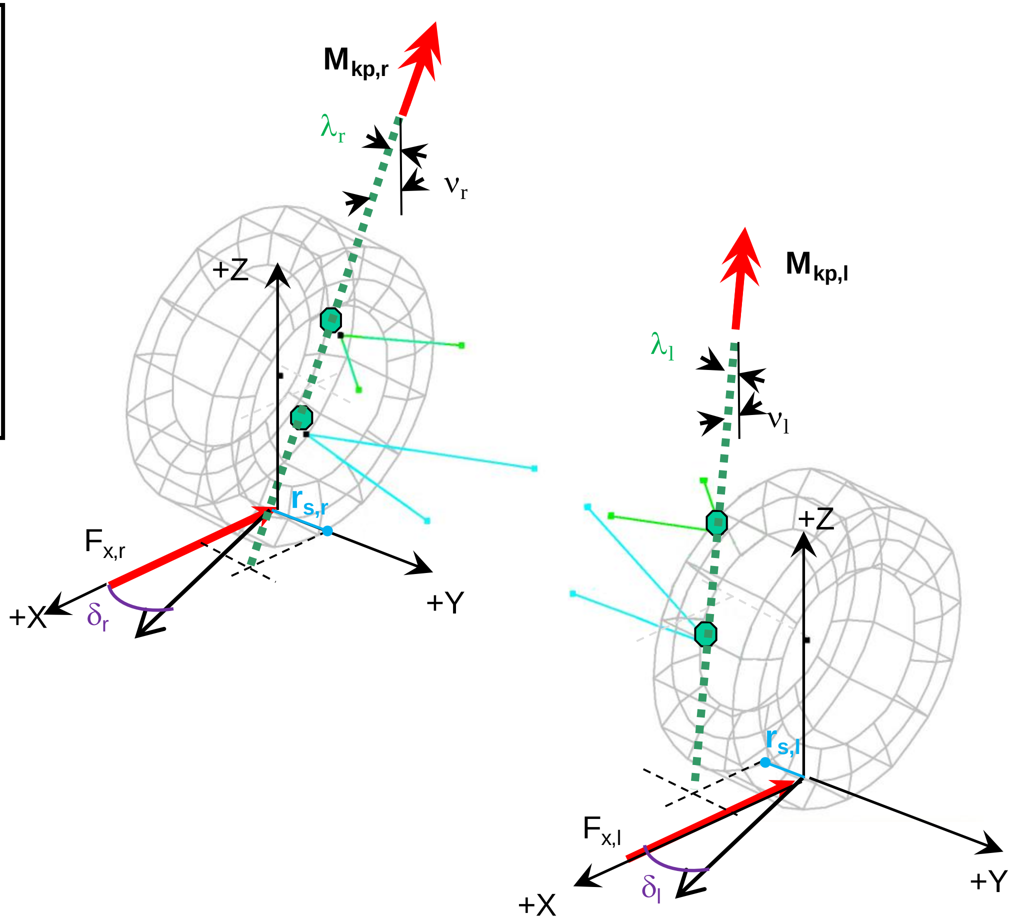
Axle Kingpin Moment from Braking Forces at CP

For the case of a braking force at the contact patch:

$$M_{kp,x} = -F_{x,r}*r_{s,r}*\cos(\lambda_r)*\cos(\delta_r)$$

$$+F_{x.l}*r_{s,l}*\cos(\lambda_l)*\cos(\delta_l)$$

.... Eq 8a



$r_{s,r}$,$r_{s,l}$ = right, left scrub radius
λ_r, λ_l = right, left kingpin inclination angle
δ = right, left road wheel steer angle

Axle Kingpin Moment from Traction Forces at the WC

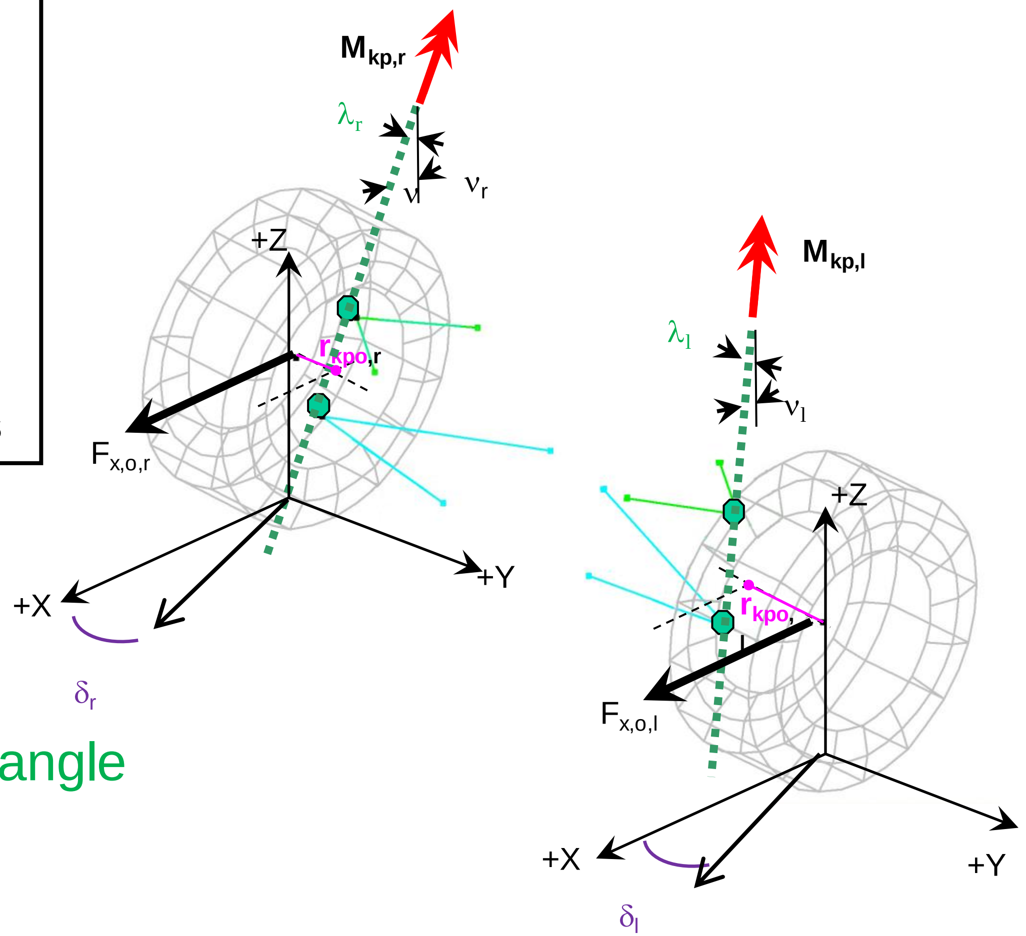
For the special case of a front wheel drive vehicle, the tractive force is applied at the wheel center:

$$M_{kp,dt} = F_{x,o,r} * r_{kp,r} * \cos(\lambda_r) * \cos(\delta_r) - F_{x,o,l} * r_{kp,l} * \cos(\lambda_l) * \cos(\delta_l)$$

…Eq 8b

Where:

$F_{\mathbf{x,o}}$ = drive torque / tire loaded radius



$r_{kp,r}$, $r_{kp,l}$ = right, left kingpin offset

λ_r, λ_l = right, left kingpin inclination angle

δ_r, δ_l = Road wheel steer angle

This equation can also be used in the case of regenerative braking, where a –Fx is applied to the WC

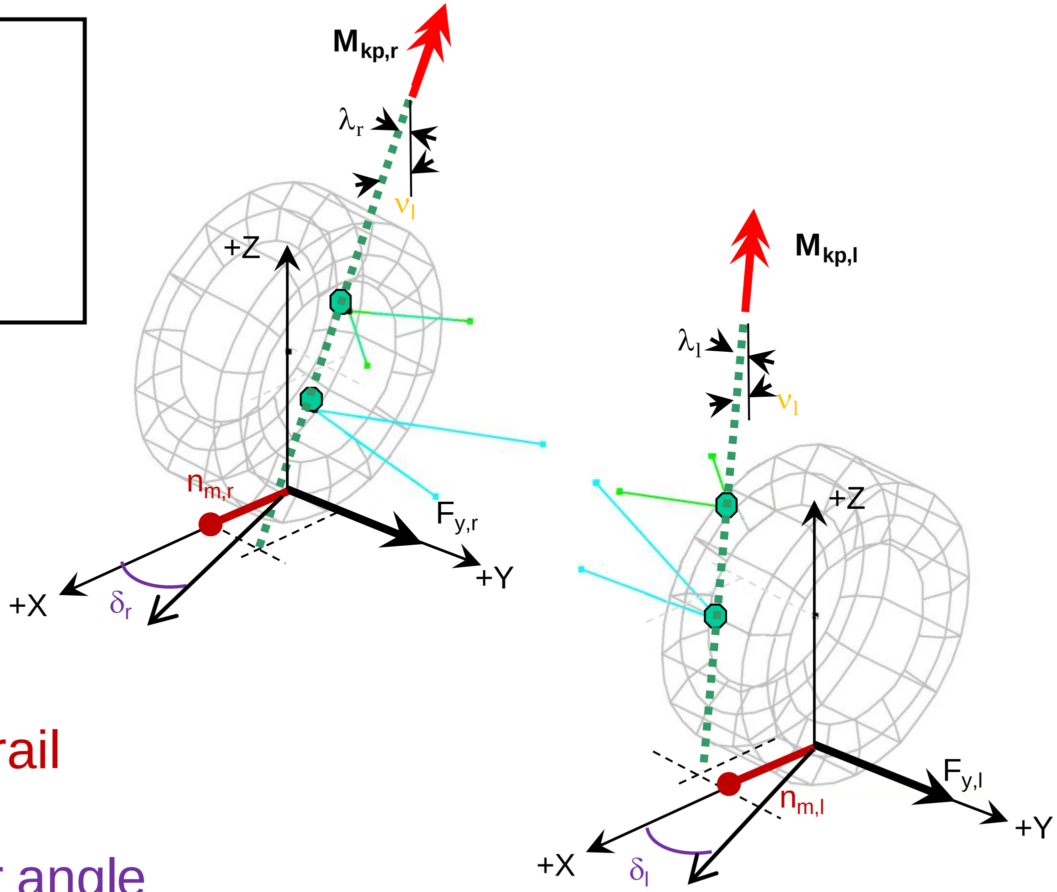
Axle Kingpin Moment from Lateral Force at CP

For a lateral force at the contact patch:

$$M_{kp,y} = -F_{y,r} * \{n_{m,r} * \cos(\nu_r)\} * \cos(\delta_r)$$

$$-F_{y,l} * \{n_{m,l} * \cos(\nu_l)\} * \cos(\delta_l)$$

….. Eq 9



$n_{m,r}$, $n_{m,l}$ = right, left mechanical trail

ν_r , $\nu_{r\ l}$ = right, left caster angle

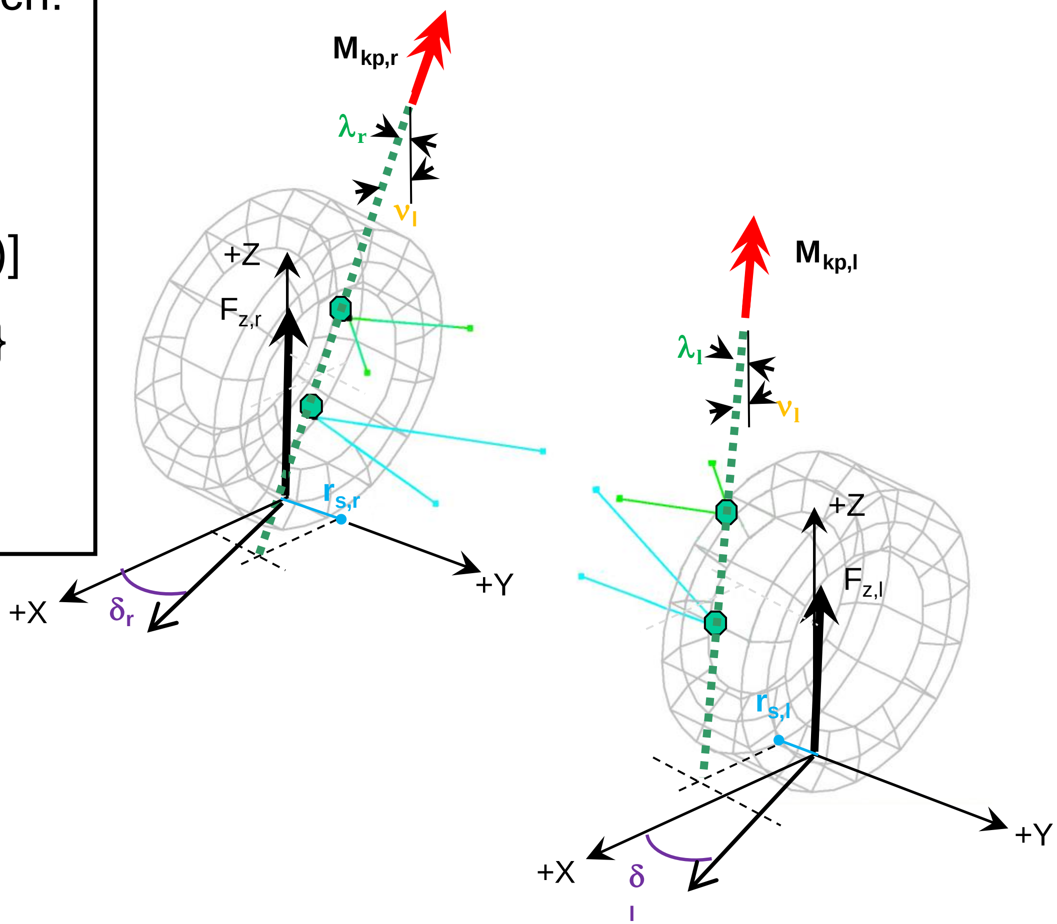
δ_r , δ_l = right, left road wheel steer angle

Axle Kingpin Moment from Vertical Force at CP

For a vertical force at the contact patch:

$$
\begin{aligned}
M_{kp,Fz} = -\{\ & [F_{z,r}*\sin(\lambda_r)]*[r_{s,r}*\sin(\delta_r)] \\
& + [F_{z,l}*\sin(\lambda_l)\}*\{r_{s,l}*\sin(\delta_l)]\ \} \\
& + \{\ [F_{z,r}*\sin(\nu_r)]*[r_{s,r}*\cos(\delta_r)] \\
& - \ [F_{z,l}*\sin(\nu_l)]*[r_{s,l}*\cos(\delta_l)]\ \}
\end{aligned}
$$

…..Eq 10

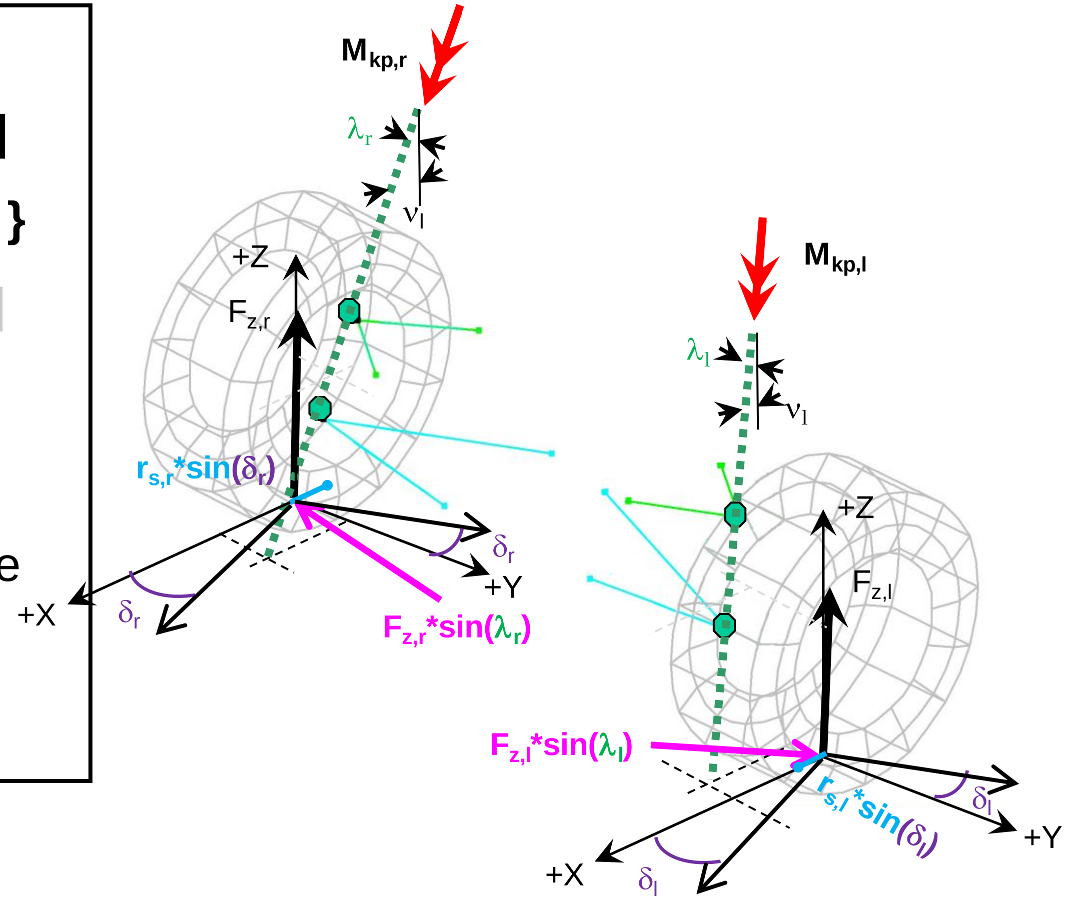


Axle Kingpin Moment from Vertical Force at CP – 1st Term

Considering the first term:

$$M_{kp,Fz} = -\{ [F_{z,r}*\sin(\lambda_r)]*[r_{s,r}*\sin(\delta_r)]$$
$$+ [F_{z,l}*\sin(\lambda_l)\}*\{r_{s,l}*\sin(\delta_l)] \}$$
$$+ \{ [F_{z,r}*\sin(\nu_r)]*[r_{s,r}*\cos(\delta_r)]$$
$$- [F_{z,l}*\sin(\nu_l)]*[r_{s,l}*\cos(\delta_l)] \}$$

KP moment on left and right are in the same direction and opposite the direction of turn. This is a self centering effect.



$F_z*\sin(\lambda)$
Component of F_z, perpendicular to kingpin axis in the Y-Z plane

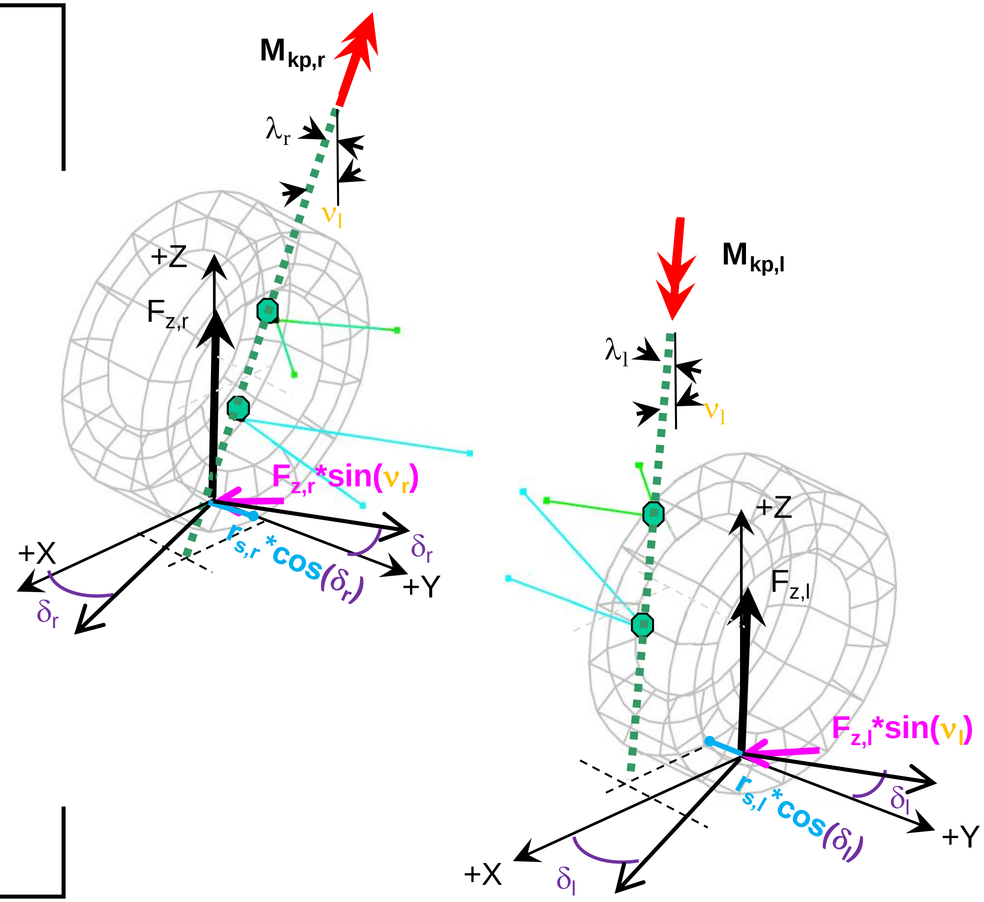
$r_{s,r}$, $r_{s,l}$ = right, left scrub radius
λ_r , λ_l = right, left kingpin inclination angle
δ_r , δ_l = right, left road wheel steer angle

Axle Kingpin Moment from Vertical Force at CP – 2nd Term

Considering the second term:

$$M_{kp,Fz} = -\{ [F_{z,r}*\sin(\lambda_r)]*[r_{s,r}*\sin(\delta_r)]$$
$$+ [F_{z,l}*\sin(\lambda_l)\}*\{r_{s,l}*\sin(\delta_l)] \}$$
$$\mathbf{+ \{ [F_{z,r}*\sin(\nu_r)]*[r_{s,r}*\cos(\delta_r)]}$$
$$\mathbf{- [F_{z,l}*\sin(\nu_l)]*[r_{s,l}*\cos(\delta_l)] \}}$$

KP moment on right and left are in the opposite direction and tend to balance through the steering linkage. An axle kingpin moment is generated when there is a difference in right and left vertical force, statically or when cornering



$F_z*\sin(\nu)$
Component of F_z, perpendicular to kingpin axis in the X-Z plane

$r_{s,r}$, $r_{s,l}$ = right, left scrub radius
ν_r , ν_l = right, left caster angle
δ_r , δ_l = right, left road wheel steer angle

Axle Kingpin Moment from Aligning Torque at CP

For an aligning torque at the contact patch, assuming some amount of tire pneumatic trail and residual aligning torque (RAT)

$$M_{kp,Mz} = M_{z,r}*\cos[(\lambda_r^2+\nu_r^2)^{0.5}]$$

$$+ M_{z,l}*\cos[(\lambda_l^2+\nu_l^2)^{0.5}] \quad \ldots\text{Eq 11}$$

Where

$$M_z = M_{z,p} + M_{z,TR} = F_y*n_p + M_{z,TR}$$

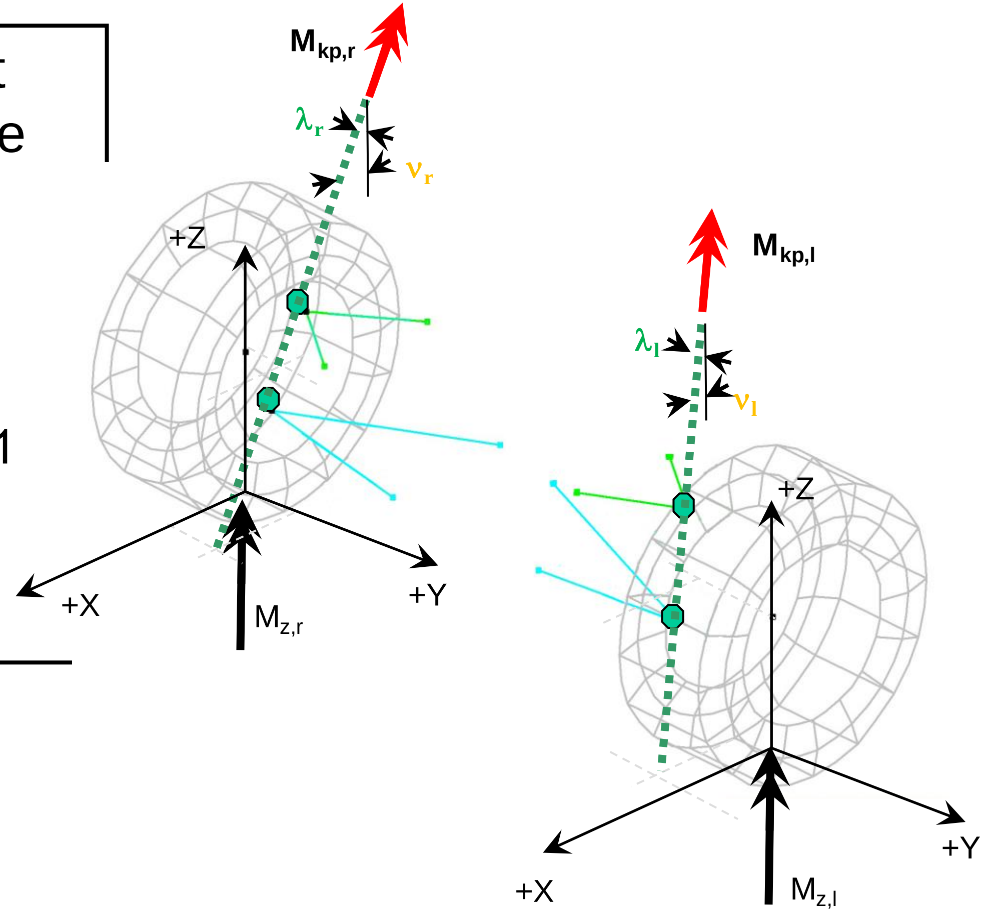
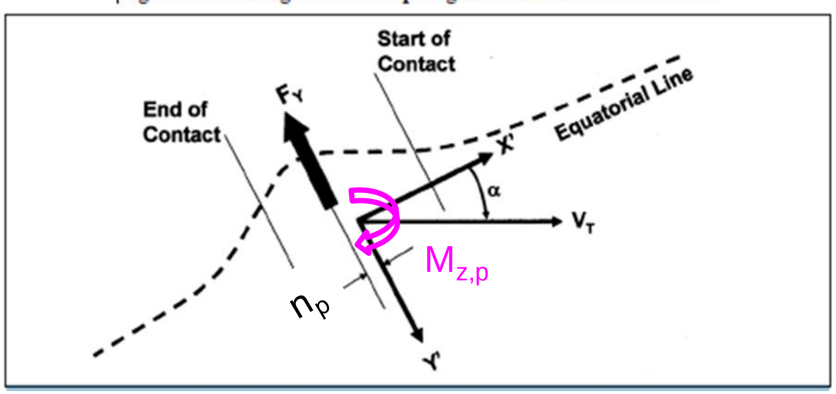


Figure 8.3: String tire at a slip angle in contact with the road



<u>Paraphrasing SAE J670e</u>

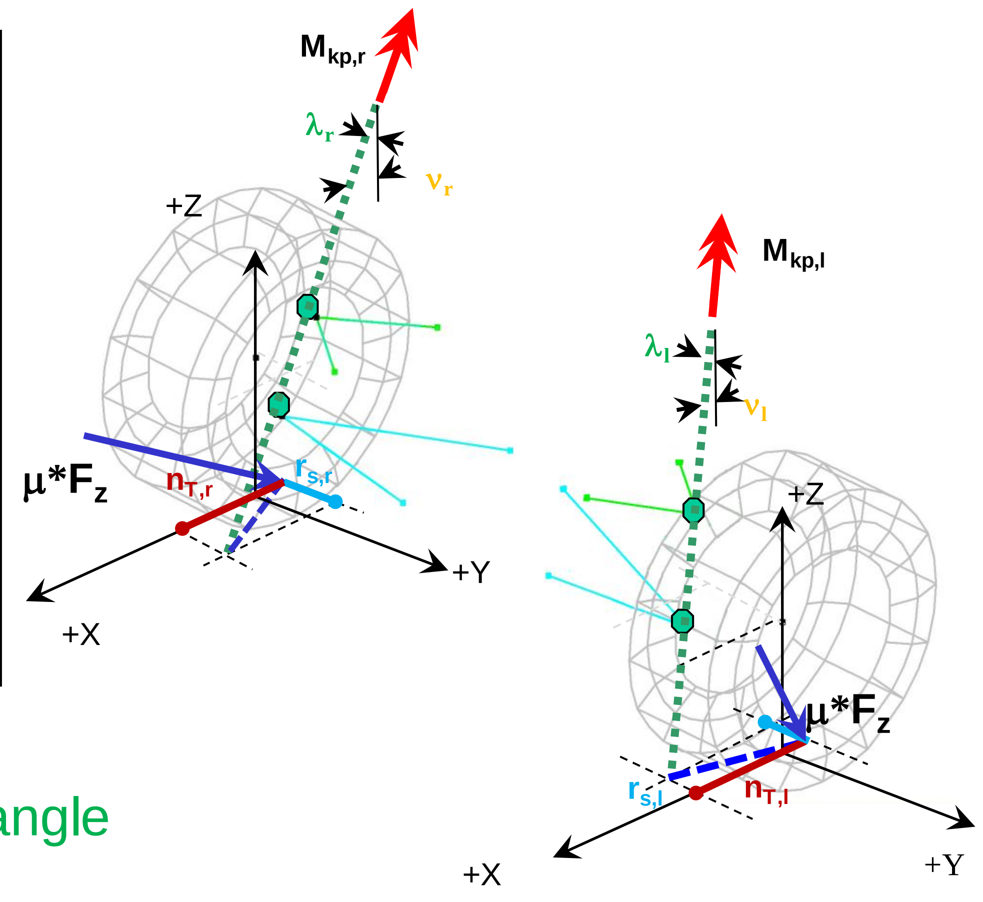
Residual Aligning Moment M_{ZTR}—The ***tire aligning moment*** at zero lateral force for a straight, free rolling tire

Axle Kingpin Moment from Steering a ‘Locked’ Tire

For the special case of steering a locked tire (aka static parking):

$$
\begin{aligned}
M_{kp,sp} = & -\mu_r{*}F_{z,r}{*}\{[r_{s,r}{*}\cos(\lambda_r)]^2 \\
& \qquad +[n_{T,r}{*}\cos(\nu_r)]^2\}^{0.5} \\
& - \mu_l{*}F_{z,l}{*}\{[r_{s,l}{*}\cos(\lambda_l)]^2 \\
& \qquad +[n_{T,l}{*}\cos(\nu_l)]^2\}^{0.5}
\end{aligned}
$$

… Eq 12



$r_{s,r}$, $r_{s,l}$ = right, left scrub radius

λ_r, λ_l = right, left kingpin inclination angle

$n_{T,r}$, $n_{T,l}$ = right, left total trail

ν_r, ν_l = right, left caster angle

Total trail (n_T) = Caster (mechanical) trail (n_m) + Pneumatic Trail (n_p)

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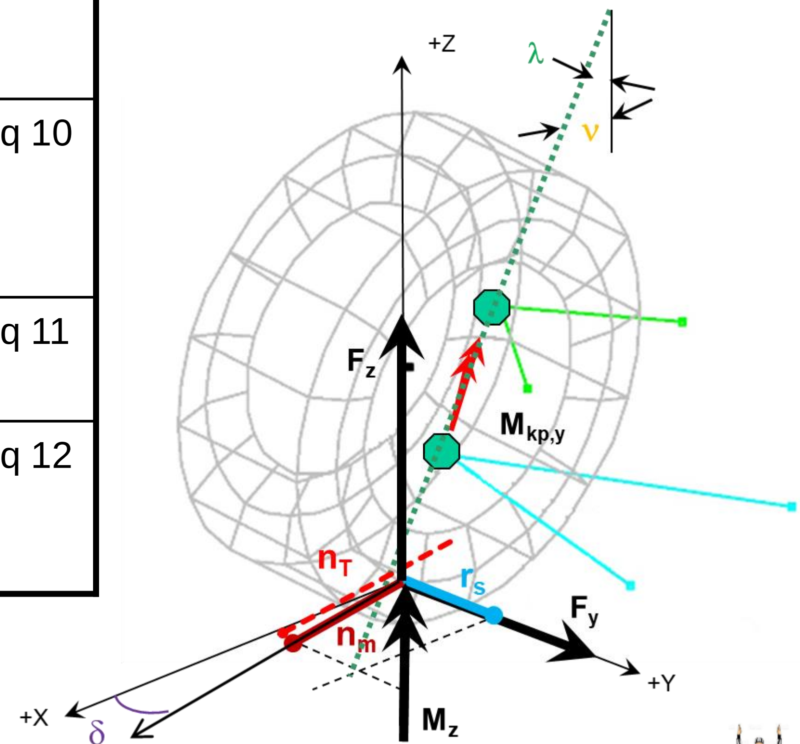
Example: Calculating the Kingpin Moment

For the specific case of a rear wheel drive vehicle with symmetric weight, symmetric front suspension, steering on a homogeneous surface, the kingpin moment equations are:

Lateral Force at Contact Patch	$M_{kp,Fy} = -(F_{y,r}+F_{y,l})*\mathbf{n_m}*\cos(\nu)*\cos(\delta)$	Eq 9
Vertical Force at Contact Patch	$M_{kp,Fz} = -(F_{z,r}+F_{z,l})*\sin(\lambda)*[\mathbf{r_s}*\sin(\delta)]$ $+(F_{z,r}-F_{z,l})*\sin(\nu)]*[\mathbf{r_s}*\cos(\delta)]$	Eq 10
Aligning Moment	$M_{kp,Mz} = (M_{z,r}+M_{z,l})*\cos[(\lambda^2+\nu^2)^{0.5}]$	Eq 11
Static Park	$M_{kp,SP} = -\mu*(F_{z,r}+F_{z,l})*$ $\{[\mathbf{r_s}*\cos(\lambda)]^2+[\mathbf{n_T}*\cos(\nu)]^2\}^{0.5}$	Eq 12



Example: Calculating the Kingpin Moment

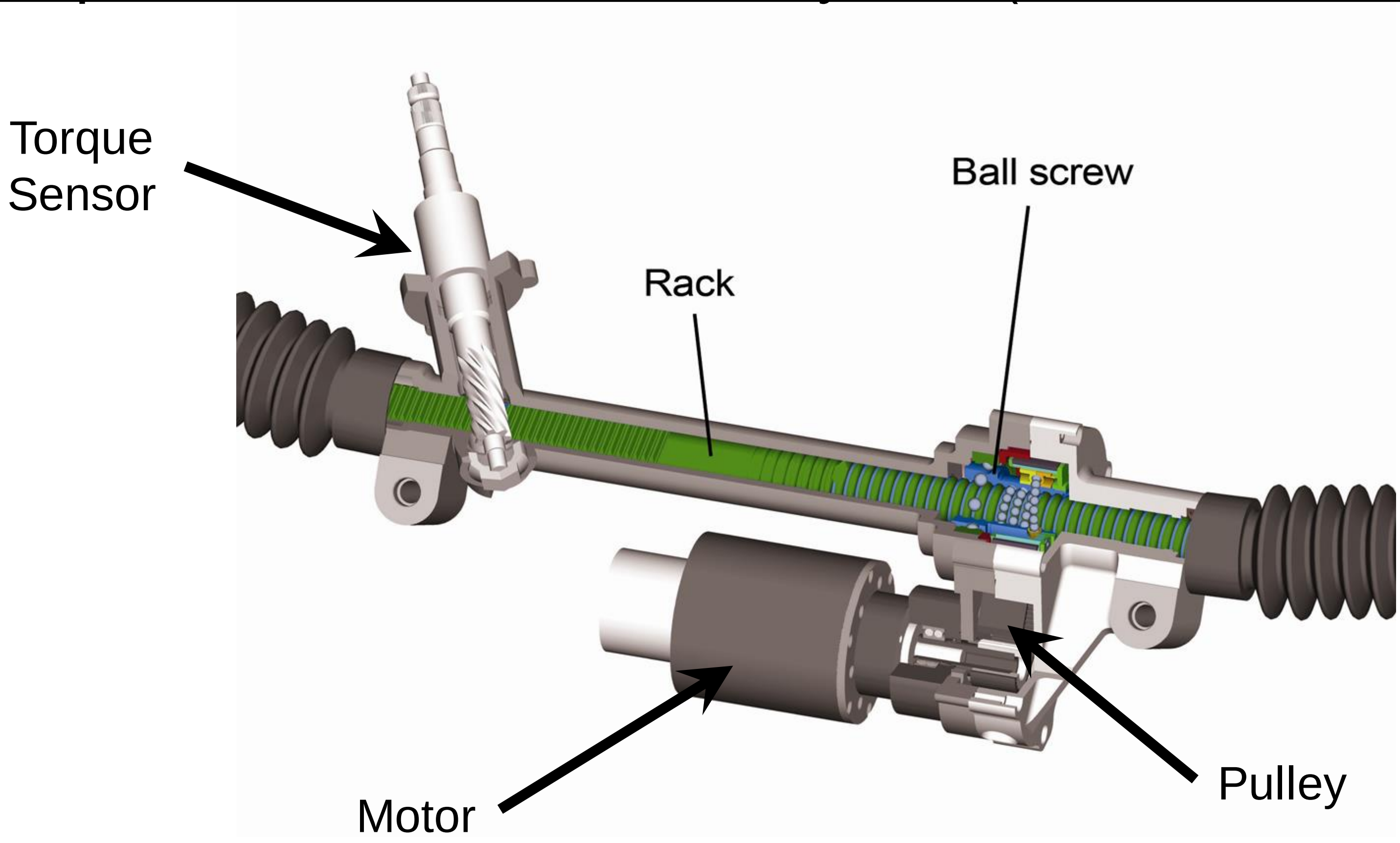
Assume we are working on a rear wheel drive sedan **with offset rack electric power steering** (OREPS). Our vehicle has the following parameters:

(yellow)	inputs
(blue)	calculated

Vehicle Information

	Symbol	**Value**	**Units**
Front Axle Vertical Load @ 2pass	W_f	10470	N
Overall steering ratio	i	14.9	:1
KPI (+= tipped inboard)	λ	14.2	deg
Caster Angle (+=tipped rearward	ν	3.6	deg
Scrub Radius (+= intersection inboard of wheel center)	r_s	-0.012	m
Static Tire Mu	μ	1	-
Mechanical (caster) trail	n_m	0.025	m
Pneumatic trail	n_p	0.030	m
Total Trail (Mechanical + Pneumatic)	n_T	0.055	m
Steer arm length	l_{sa}	0.143	m

Example of an offset-rack EPS system (aka “belt drive)

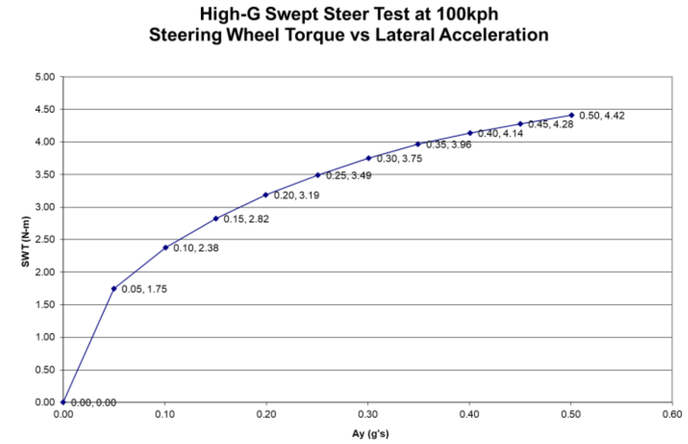


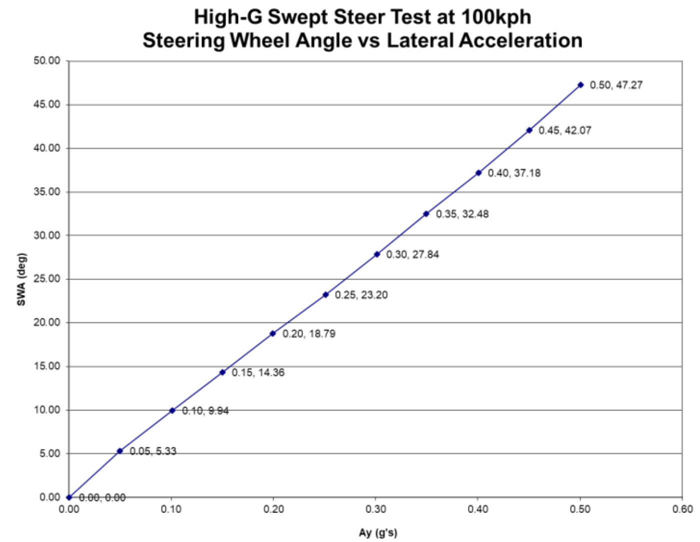
Adapted from article “High Precision ball screw drive improves driver handling and fuel consumption on VW Tiguan” http://www.schaeffler.co.uk/content.schaeffler.co.uk/en/press/press-releases/press-details.jsp?id=2549136

Example: Calculating the Kingpin Moment

Suppose we have the following steering performance targets:

Ay (g's)	θ_{sw} (deg)	T_{sw} (Nm)
0.00	0.00	0.00
0.05	5.33	1.75
0.10	9.94	2.38
0.15	14.36	2.82
0.20	18.79	3.19
0.25	23.20	3.49
0.30	27.84	3.75
0.35	32.48	3.96
0.40	37.18	4.14
0.45	42.07	4.28
0.50	47.27	4.42
Static Parking Effort around center (OC)		2.5





Example: Calculating the Kingpin Moment

For this analysis, we will make the following assumptions:

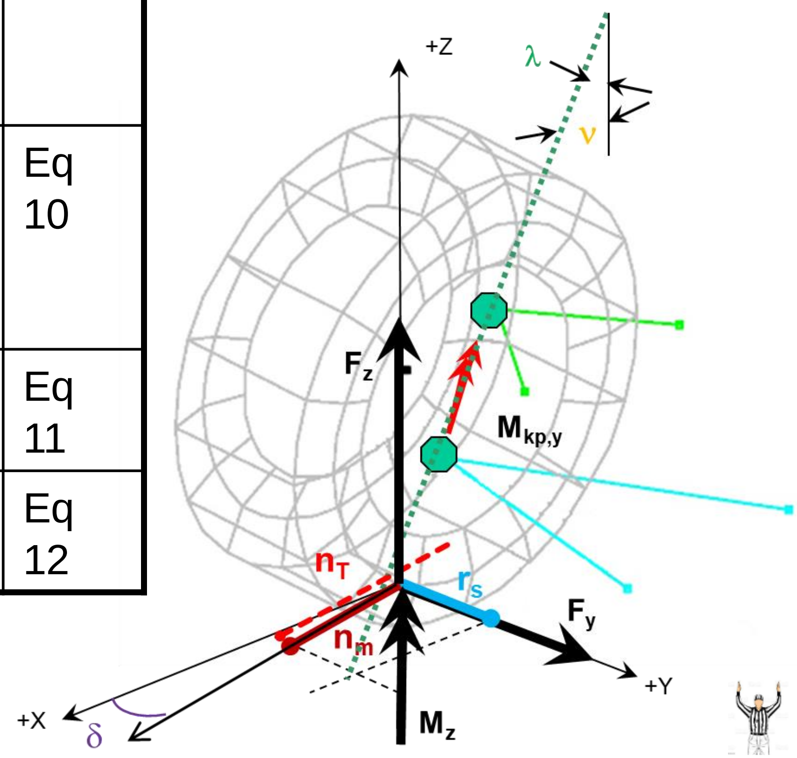
- Symmetric front suspension geometry
- Scrub radius, mechanical trail, kingpin inclination and caster angle are constant
- Homogeneous surface
- Free rolling front tires; $F_x = 0$
- No tire residual aligning torque; $M_{z,RAT} = 0$
- Negligible lower steering friction; $F_{r,p} = F_{sa,y}$

Example: Calculating the Kingpin Moment

Using the following equations, we calculate the total moment about the kingpin axis due to forces and moments at the contact patch.

Road wheel angle as function of steering wheel angle: $\delta = \theta_{sw} / i$
Lateral force at the contact patch: $F_y = W_f * A_y$

Lateral Force at Contact Patch	$M_{kp,Fy} = -(F_{y,r}+F_{y,l})*n_m*\cos(\nu)*\cos(\delta)$	Eq 9
Vertical Force at Contact Patch	$M_{kp,Fz} = -(F_{z,r}+F_{z,l})*\sin(\lambda)*[r_s*\sin(\delta)] +(F_{z,r}-F_{z,l})*\sin(\nu)]*[r_s*\cos(\delta)]$	Eq 10
Aligning Moment	$M_{kp,Mz} = (M_{z,r}+M_{z,l})*\cos[(\lambda^2+\nu^2)^{0.5}]$	Eq 11
Static Park	$M_{kp,SP} = -\mu*(F_{z,r}+F_{z,l})* \{[r_s*\cos(\lambda)]^2+[n_T*\cos(\nu)]^2\}^{0.5}$	Eq 12



Total kingpin moment:

$$M_{KP, Driving} = M_{KP,Fy} + M_{KP,Fz} + M_{KP,Mz} \; ; M_{KP,SP}$$

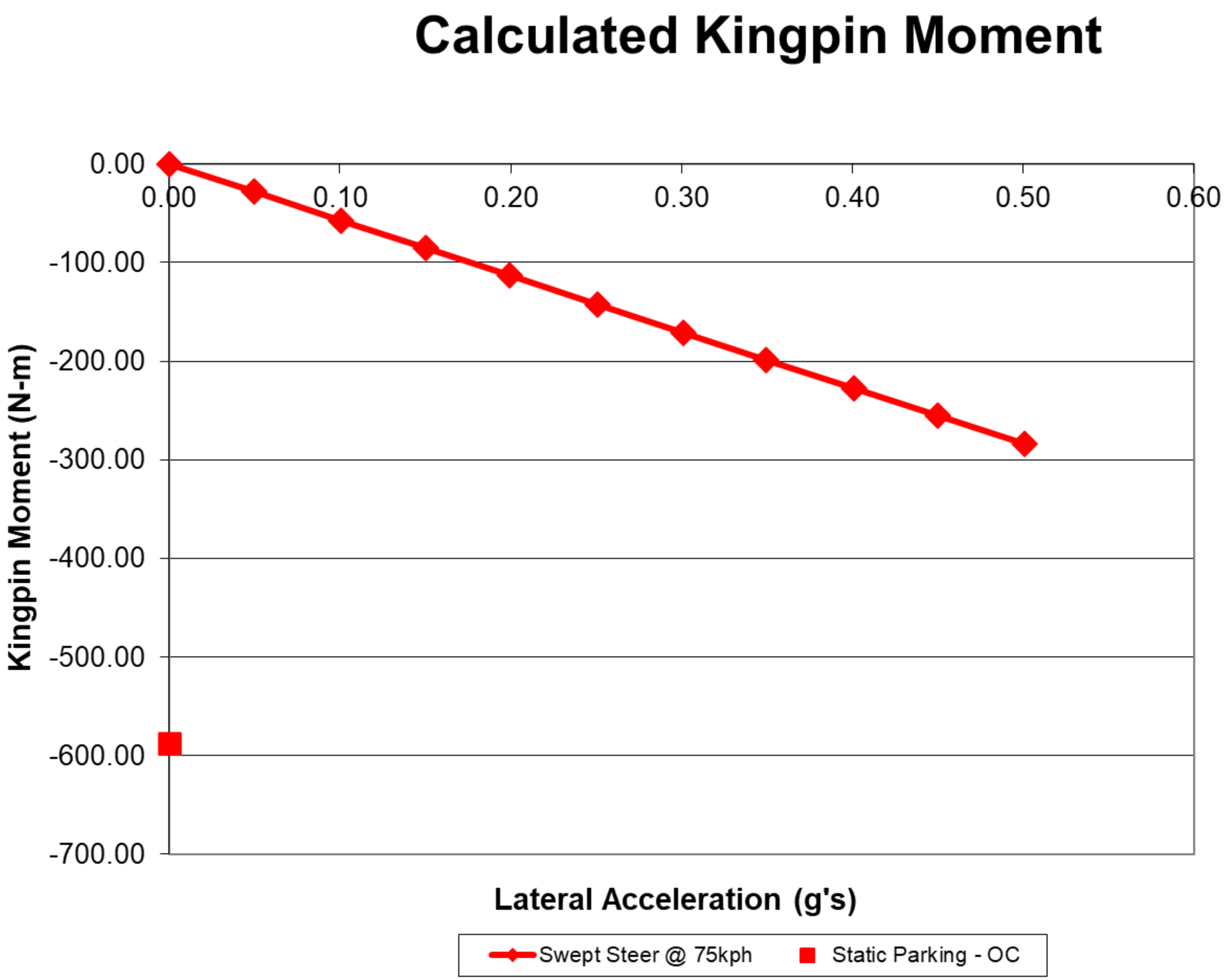
Tim Drotar/tdrotar08@gmail.com

Using Excel:

Step 1: Calculate the total moment about KP axis

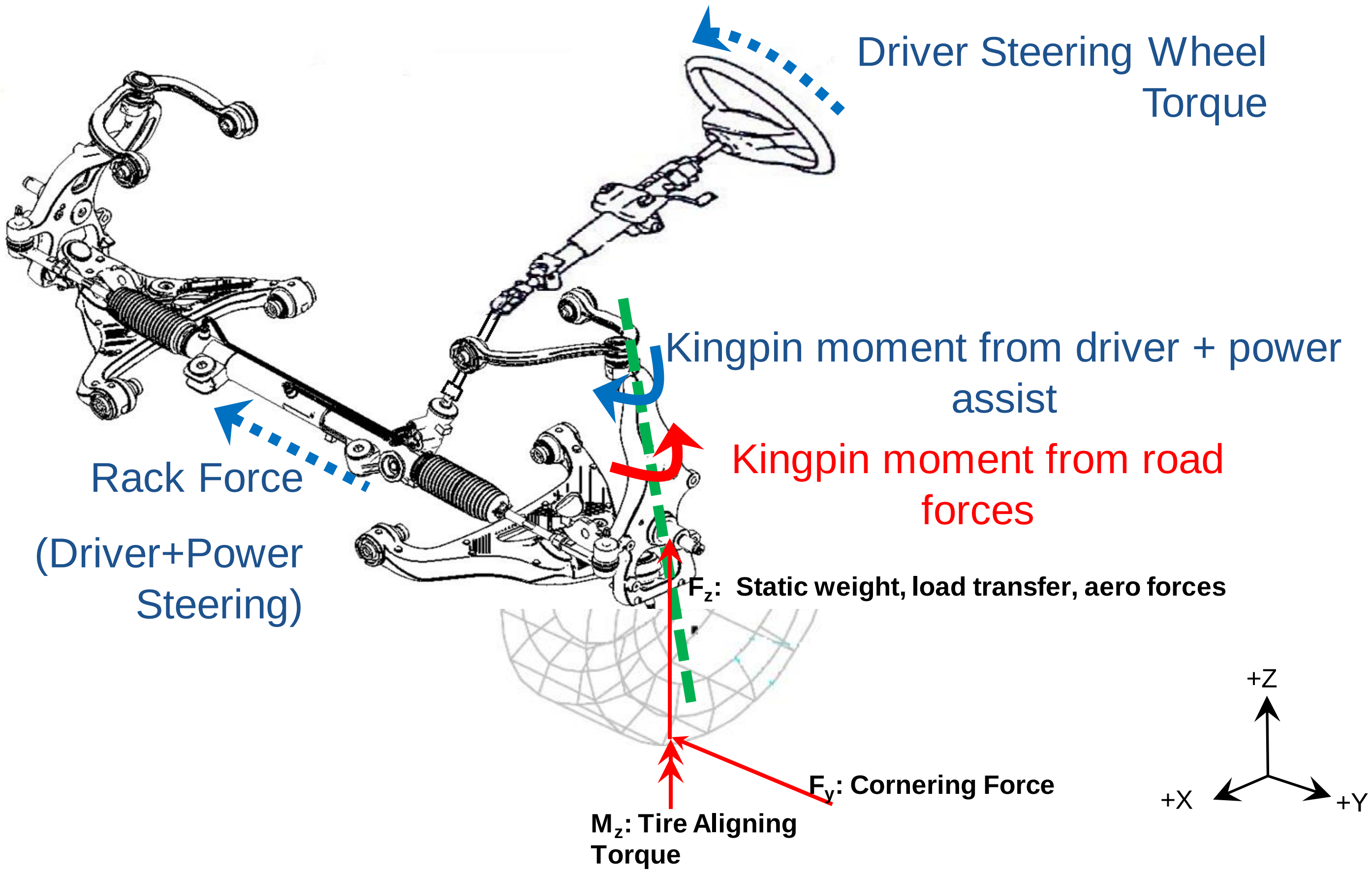
Given		Equation =>	Θ_{sw}/i	W_f*A_y	Module 2 Equ. 9	Module 2 Equ. 10	Module 2 Equ. 11	Module 2 Equ. 12	$\Sigma M_{kp,n}$
A_y (g's)	Θ_{sw} (deg)	T_{sw} (Nm)	δ_{rw} (deg)	$F_{y,t}$ (N)	$M_{KP,Fy}$ (Nm)	$M_{KP,Fz}$ (Nm)	$M_{KP,Mz}$ (Nm)	$M_{KP,SP}$ (Nm)	**M_{KP} (Nm)**
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0		**0.00**
0.05	4.21	1.81	0.28	522.13	-13.0	-0.10	-15.2		**-28.28**
0.10	7.70	2.45	0.52	1056.58	-26.4	-0.23	-30.7		**-57.26**
0.15	10.98	2.91	0.74	1574.09	-39.3	-0.36	-45.7		**-85.32**
0.20	14.25	3.32	0.96	2088.96	-52.1	-0.49	-60.6		**-113.23**
0.25	17.68	3.68	1.19	2627.96	-65.6	-0.62	-76.3		**-142.45**
0.30	21.08	3.99	1.41	3151.76	-78.6	-0.75	-91.5		**-170.84**
0.35	24.45	4.23	1.64	3660.37	-91.3	-0.87	-106.2		**-198.40**
0.40	28.12	4.45	1.89	4196.52	-104.6	-0.99	-121.8		**-227.45**
0.45	31.80	4.64	2.13	4714.02	-117.5	-1.11	-136.8		**-255.47**
0.50	35.72	4.81	2.40	5242.47	-130.7	-1.22	-152.2		**-284.07**
Parking - on center		2.50						-587.48	**-587.48**

Example: Calculating the Kingpin Moment



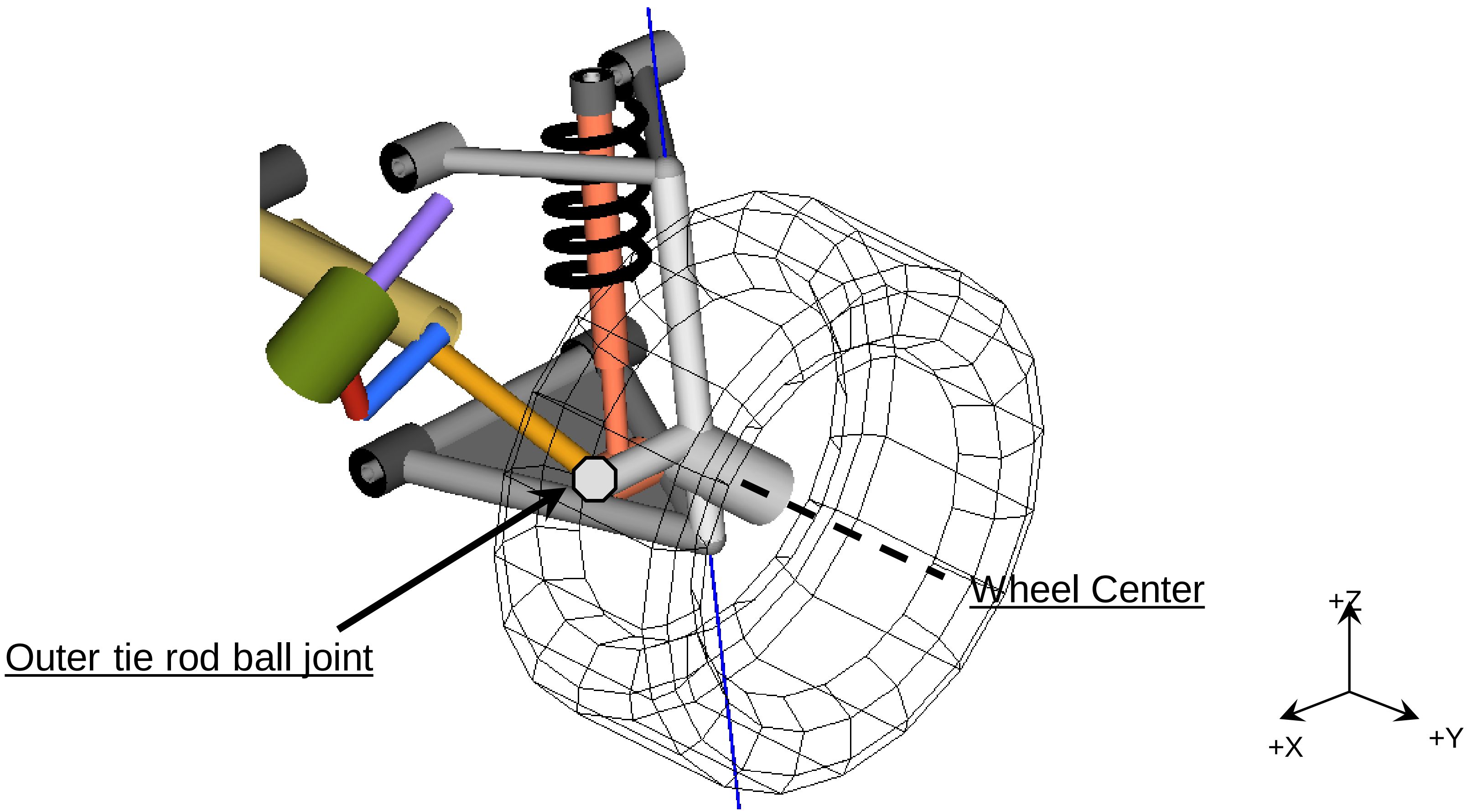
The Kingpin Moment

To keep the vehicle system in a state of equilibrium, a certain amount of rack force is required to generate a kingpin moment that is equal in magnitude and opposite in direction.



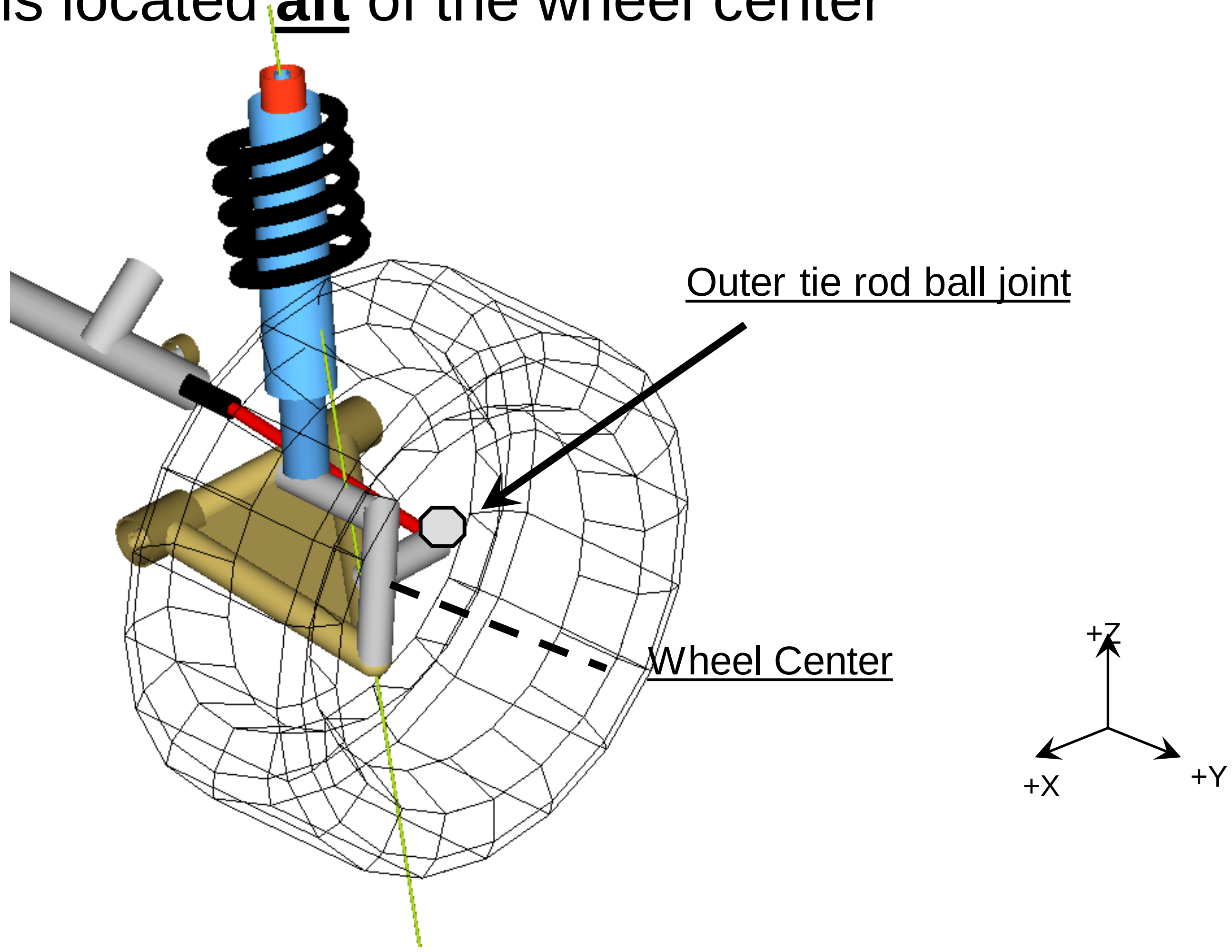
The kingpin moment is related to rack force by the **steer arm.**

A steering system is referred to as **front steer** when the outer tie rod ball joint is located **forward** of the wheel center



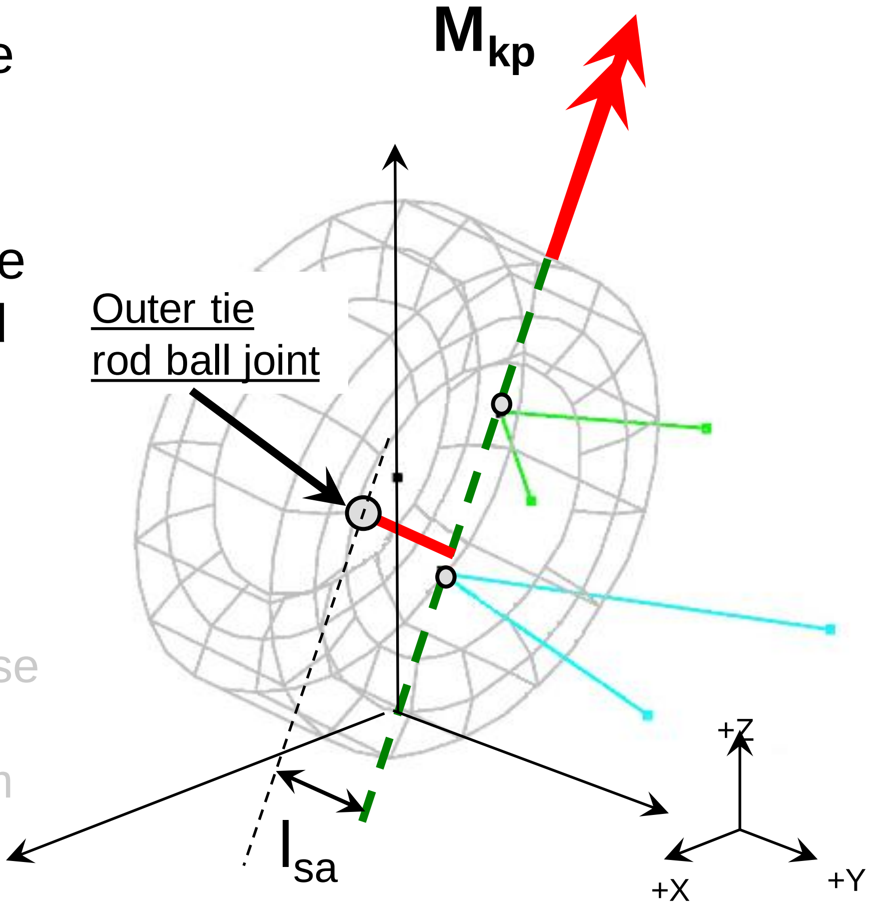
The Steer Arm

A steering system is referred to as **rear steer** when the outer tie rod ball joint is located **aft** of the wheel center



The steer arm (l_{sa}) is:

- The lever arm on which the force from the tie rod counteracts the kingpin moment.
- The normal distance between the kingpin axis and the outer tie rod ball joint
- Tends to be a constant length throughout the range of vertical wheel travel and steer
  - One notable exception is the case of a double ball joint front suspension, where the steer arm length changes with vertical wheel travel and steer

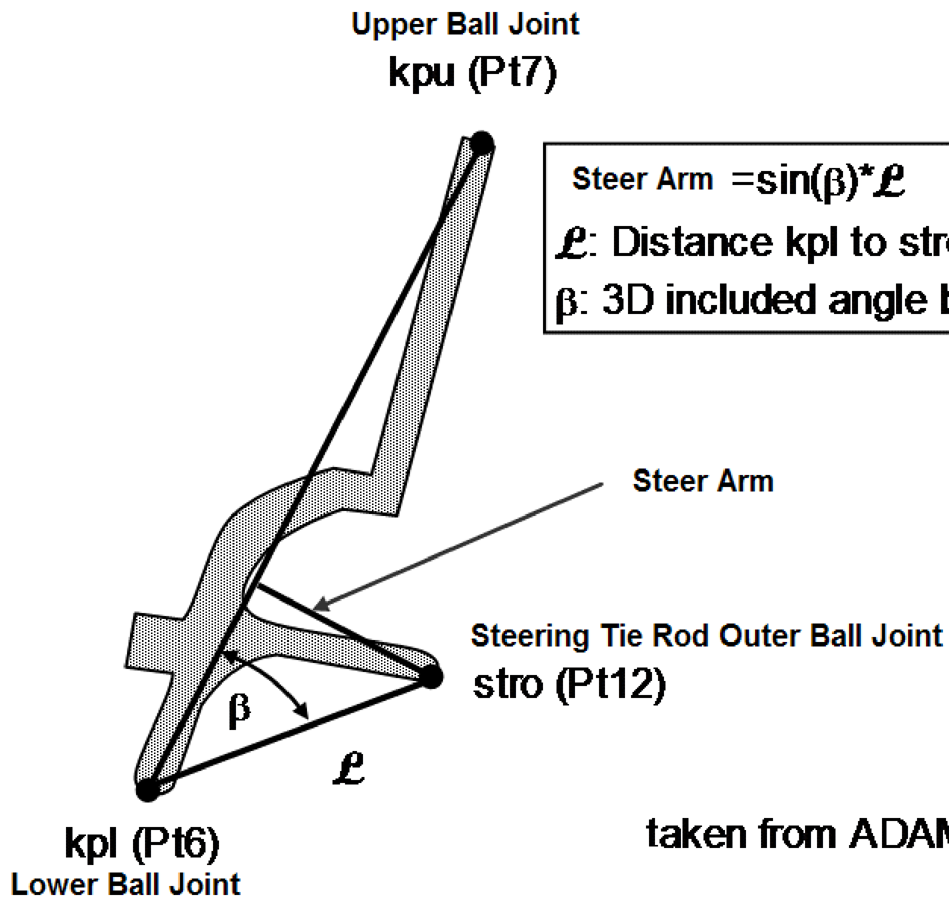


Steer arm length calculation in ADAMS

Steer Arm $=\sin(\beta)*\mathcal{L}$

$\mathcal{L}$: Distance kpl to stro

β: 3D included angle between kpl-kpu and kpl-stro



taken from ADAMS/Chassis (ratio_compute.py)

The Steer Arm

Steer arm length from steering overall ratio and rack and pinion steering gear ratio

$$L_{sa} = \frac{1}{2\pi} * i * G_r * \cos\lambda * \cos\nu$$

In books and papers, you will see this equation without the 2 ‘cos’ terms. Adding the 2 ‘cos’ terms converts the steer arm from being about a vertical axis perpendicular to the ground to being, correctly, normal to the kingpin axis

Example:

Steering overall ratio	i = 17:1
R&P steering gear ratio	G_r= 54.5mm/rev
Kingpin inclination angle	λ = 12 deg
Caster angle	ν = 5 deg

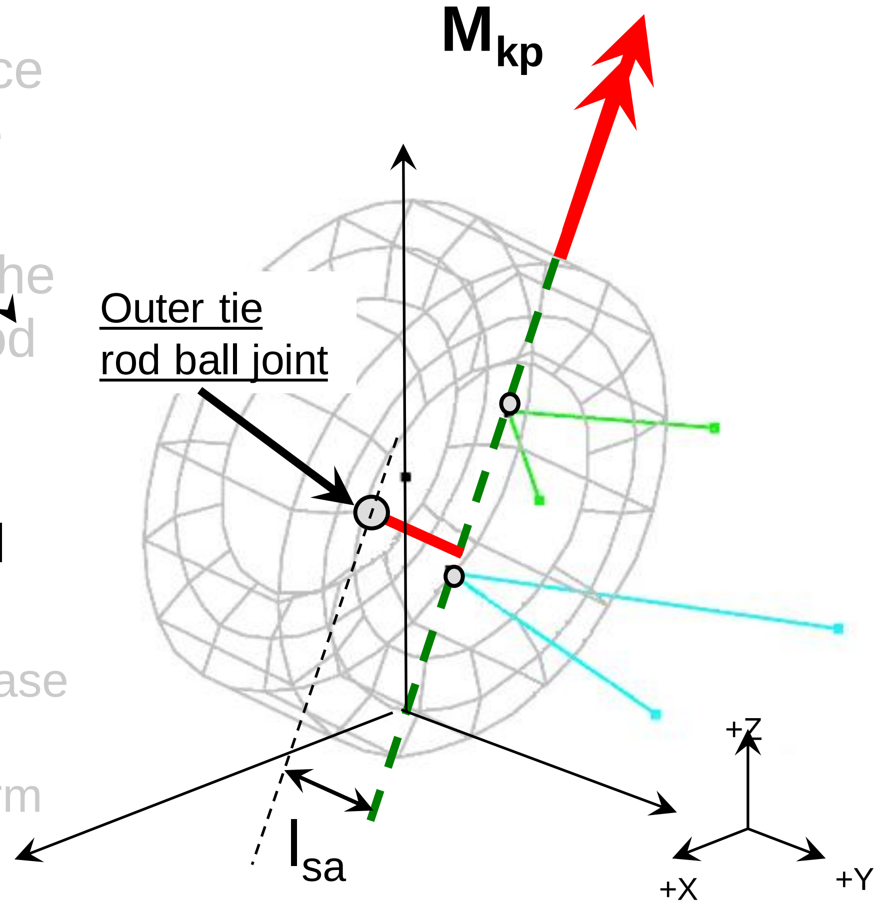
L_{sa}= (1/6.28)*17*54.5*cos(12)*cos(5) = 143.7mm

.

The Steer Arm

The steer arm (l_{sa}) is:

- The lever arm on which the force from the tie rod counteracts the kingpin moment.
- The normal distance between the kingpin axis and the outer tie rod ball joint
- Tends to be a constant length throughout the range of vertical wheel travel and steer
  - One notable exception is the case of a double ball joint front suspension, where the steer arm length changes with vertical wheel travel and steer

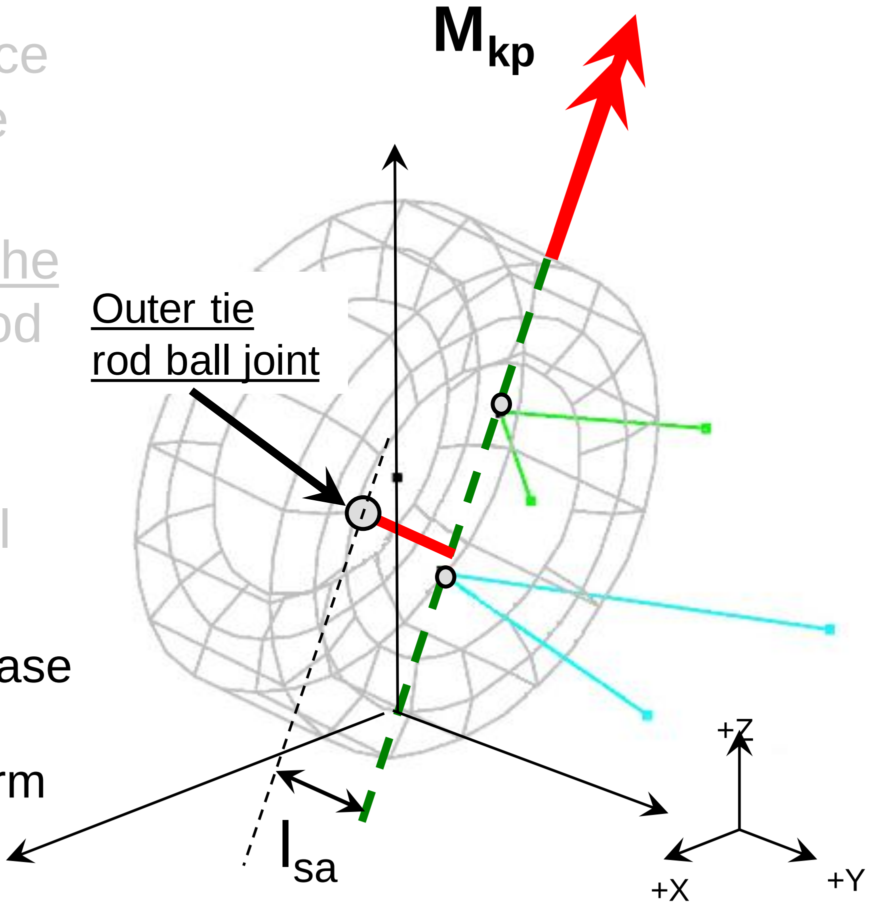


Exemplar steer arm length vs road wheel angle for a non-double ball joint front suspension

ADAMS MBD simulation

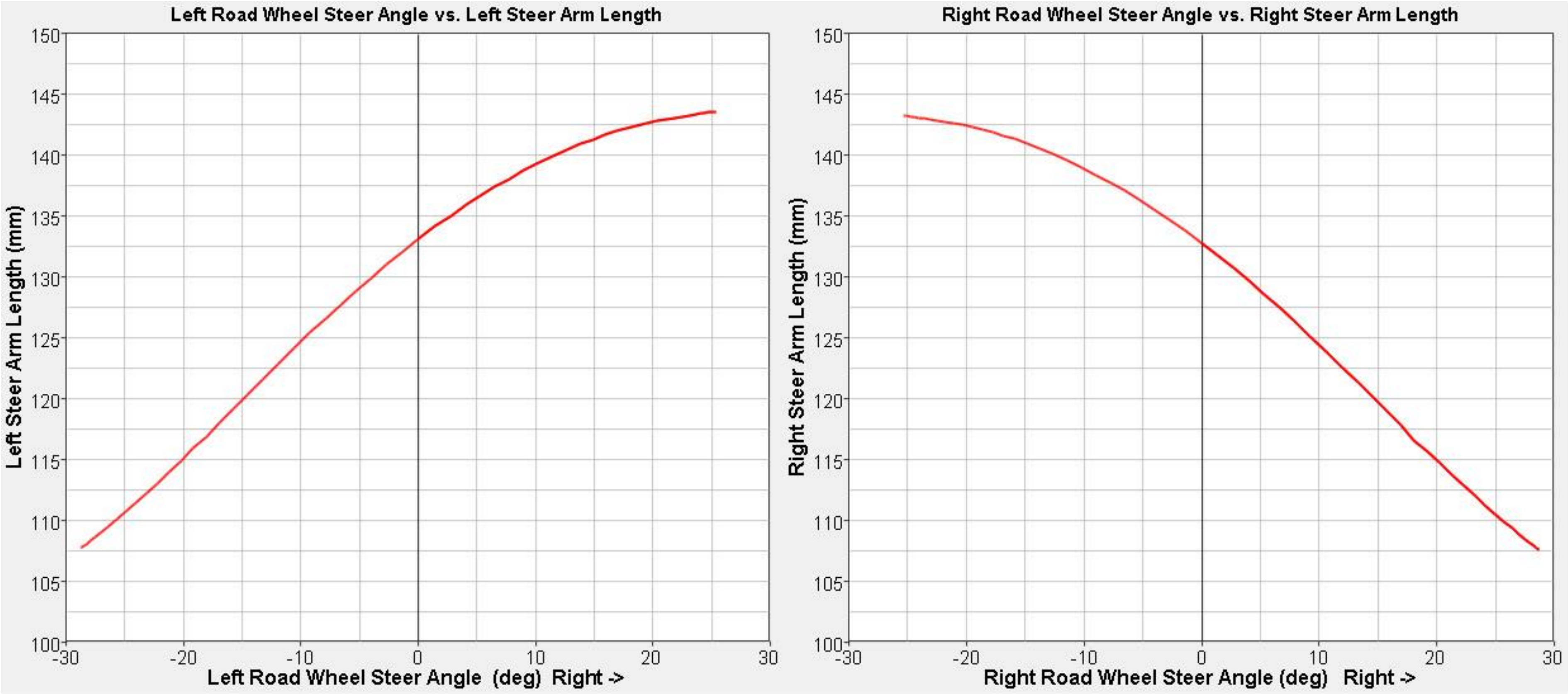
The steer arm (l_{sa}) is:

- The lever arm on which the force from the tie rod counteracts the kingpin moment.
- The normal distance between the kingpin axis and the outer tie rod ball joint
- Tends to be a constant length throughout the range of vertical wheel travel and steer
  - One notable exception is the case of a double ball joint front suspension, where the steer arm length changes with vertical wheel travel and steer



The Steer Arm

Exemplar steer arm length vs road wheel angle for a double ball joint front suspension



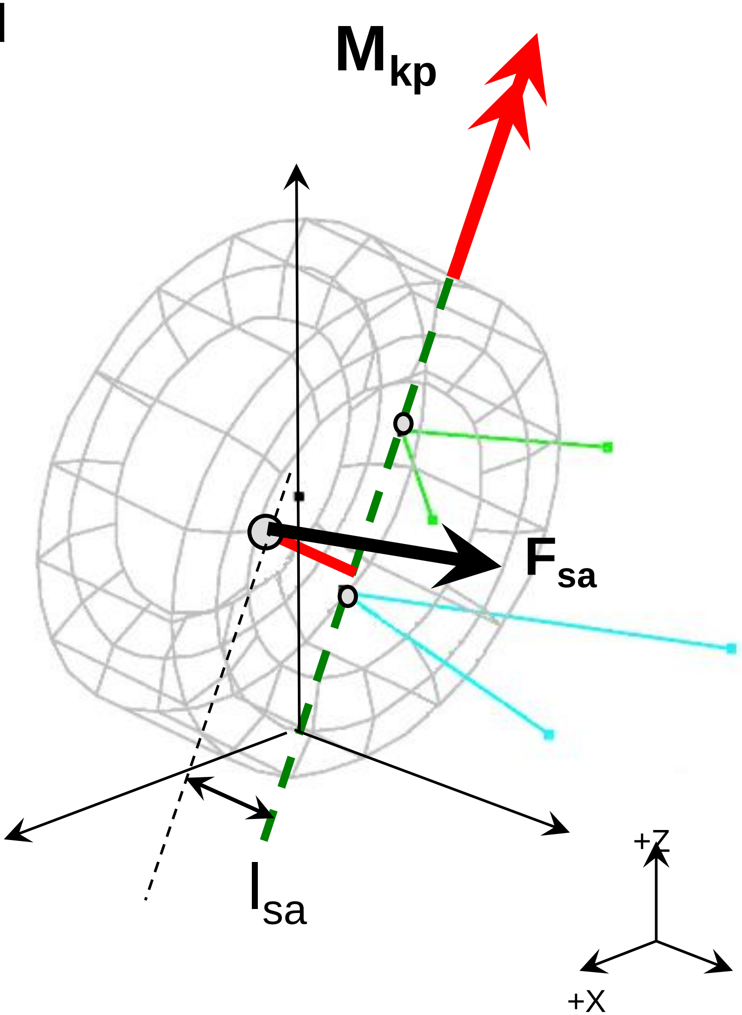
ADAMS MBD simulation

The vector force at the steer arm is defined as:

$F_{sa} = M_{kp} / l_{sa}$ ….. Equ 13

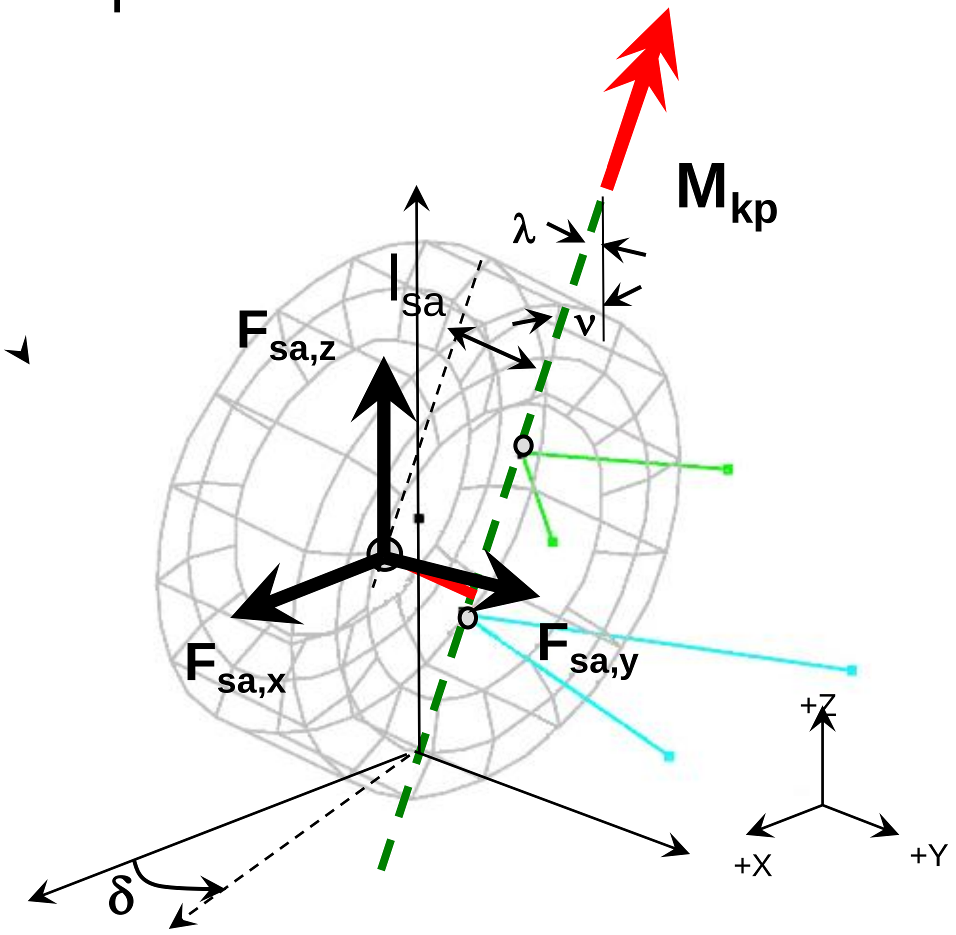
Considering the components of the force:

- Magnitude of force changes with steer angle
- The "y" component is the major component of force and defines the amount of rack force required to maintain equilibrium
- The "x" and "z" components are much smaller in magnitude, but can have a significant effect on the amount of friction in the lower steering system



If we assume that the initial plan view angle of the steer arm is zero (i.e parallel to x axis), the component forces become:

$F_{sa,y} = M_{kp} / [l_{sa}*\cos(\lambda)*\cos(\delta)]$ .. Eq 14



Example: Converting KP Moment to Rack Force

Using the following equations, we calculate the total force required by the steering rack to counteract the forces and moments at the contact patch

Vector force at the steer arm:

$F_{sa} = M_{kp} / l_{sa}$ (eq. 13)

Y-component of the vector force at the steer arm:

$Fsa{,}y = M_{kp} / [l_{sa} * \cos(\lambda) * \cos(\delta)]$ (eq. 14)

Since we are neglecting lower steering friction, all of the force from the power steering rack will be used to counteract the steer arm force. Therefore,

$F_{r,p} = -F_{sa,y}$

Example: Converting KP Moment to Rack Force

(yellow)	inputs
(blue)	calculated

Vehicle Information

	Symbol	Value	Units
Front Axle Vertical Load @ 2pass	W_f	10470	N
Overall steering ratio	i	14.9	:1
KPI (+= tipped inboard)	λ	14.2	deg
Caster Angle (+=tipped rearward	ν	3.6	deg
Scrub Radius (+= intersection inboard of wheel center)	r_s	-0.012	m
Static Tire Mu	μ	1	-
Mechanical (caster) trail	n_m	0.025	m
Pneumatic trail	n_p	0.030	m
Total Trail (Mechanical + Pneumatic)	n_T	0.055	m
Steer arm length	l_{sa}	0.143	m

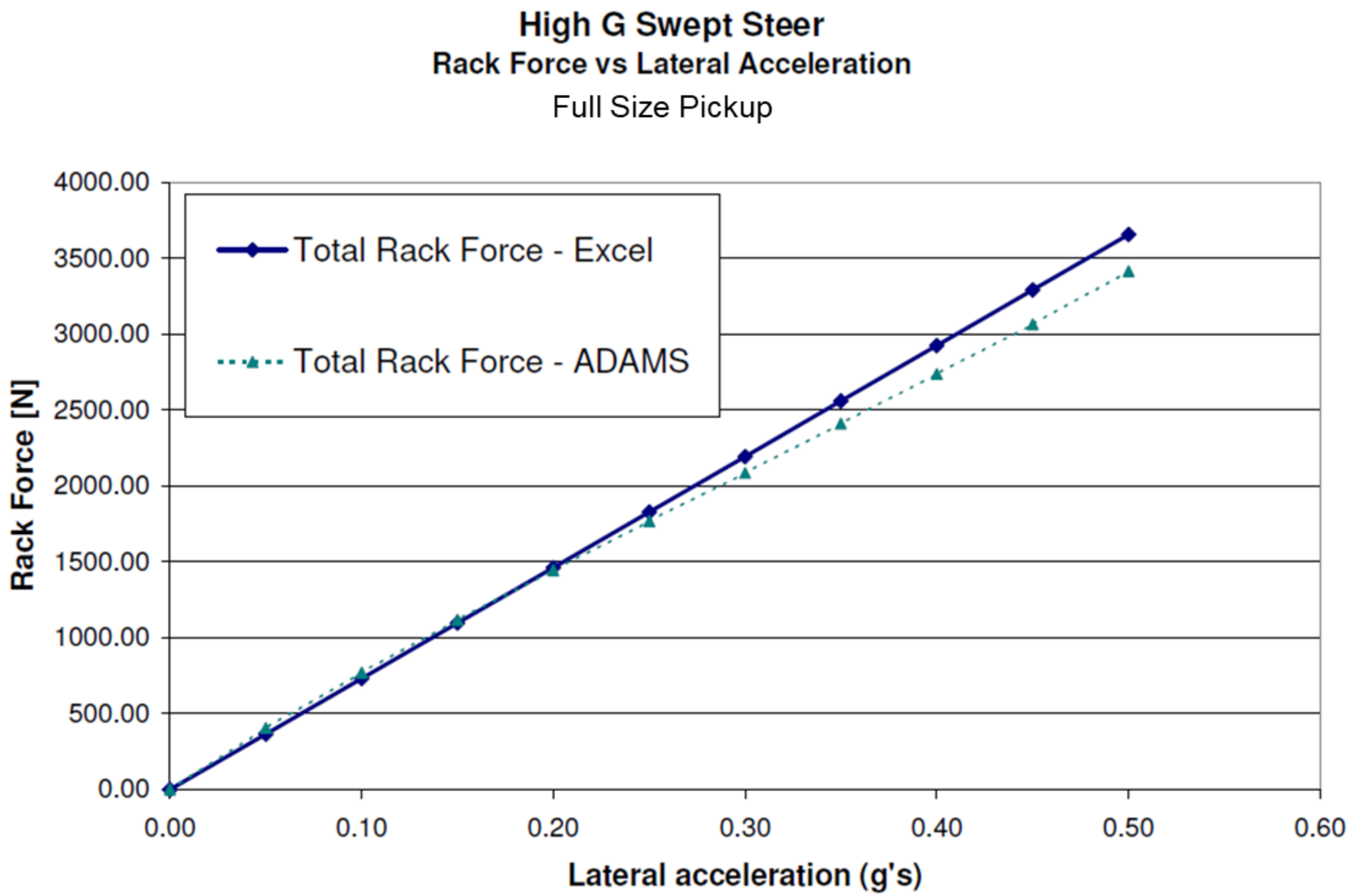
Example: Converting KP Moment to Rack Force

Using Excel:

					Step 1: Calculate the total moment about KP axis						Step 2: Calculate total rack force	
Equation => / **Given**			Θ_{sw}/i	$W_f{*}A_y$	Module 2 Equ. 9	Module 2 Equ. 10	Module 2 Equ. 11	Module 2 Equ. 12	$\Sigma M_{kp,n}$	Module 2 Equ. 13	Module 2 Equ. 15	$\mathbf{F_{r,p} = -F_{sa,y}}$
A_y (g's)	Θ_{sw} (deg)	T_{sw} (Nm)	δ_{rw} (deg)	$F_{y,t}$ (N)	$M_{KP,Fy}$ (Nm)	$M_{KP,Fz}$ (Nm)	$M_{KP,Mz}$ (Nm)	$M_{KP,SP}$ (Nm)	M_{KP} (Nm)	F_{sa} (N)	$F_{sa,y}$ (N)	$\mathbf{F_{r,p}}$ **(N)**
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0		0.00	0.00	0.00	**0.00**
0.05	4.21	1.81	0.28	522.13	-13.0	-0.10	-15.2		-28.28	-197.77	-204.00	**204.00**
0.10	7.70	2.45	0.52	1056.58	-26.4	-0.23	-30.7		-57.26	-400.40	-413.03	**413.03**
0.15	10.98	2.91	0.74	1574.09	-39.3	-0.36	-45.7		-85.32	-596.62	-615.47	**615.47**
0.20	14.25	3.32	0.96	2088.96	-52.1	-0.49	-60.6		-113.23	-791.83	-816.90	**816.90**
0.25	17.68	3.68	1.19	2627.96	-65.6	-0.62	-76.3		-142.45	-996.17	-1027.78	**1027.78**
0.30	21.08	3.99	1.41	3151.76	-78.6	-0.75	-91.5		-170.84	-1194.71	-1232.73	**1232.73**
0.35	24.45	4.23	1.64	3660.37	-91.3	-0.87	-106.2		-198.40	-1387.44	-1431.75	**1431.75**
0.40	28.12	4.45	1.89	4196.52	-104.6	-0.99	-121.8		-227.45	-1590.54	-1641.55	**1641.55**
0.45	31.80	4.64	2.13	4714.02	-117.5	-1.11	-136.8		-255.47	-1786.50	-1844.08	**1844.08**
0.50	35.72	4.81	2.40	5242.47	-130.7	-1.22	-152.2		-284.07	-1986.51	-2050.91	**2050.91**
Parking - on center		2.50						-587.48	-587.48	-4108.25	-4237.71	**4237.71**

For this vehicle to maintain a steady state turn at 0.3g, the driver + power steering system must provide a **total rack force of 1232N**.

First Principals Rack Force Correlation to MBD



For next time….

We now know how much rack force it takes to turn the road wheels during parking and steady state cornering.

If we set targets for how much effort (steering wheel torque) we want the driver to provide under these conditions, we can calculate how much assist force is required from the power steering system

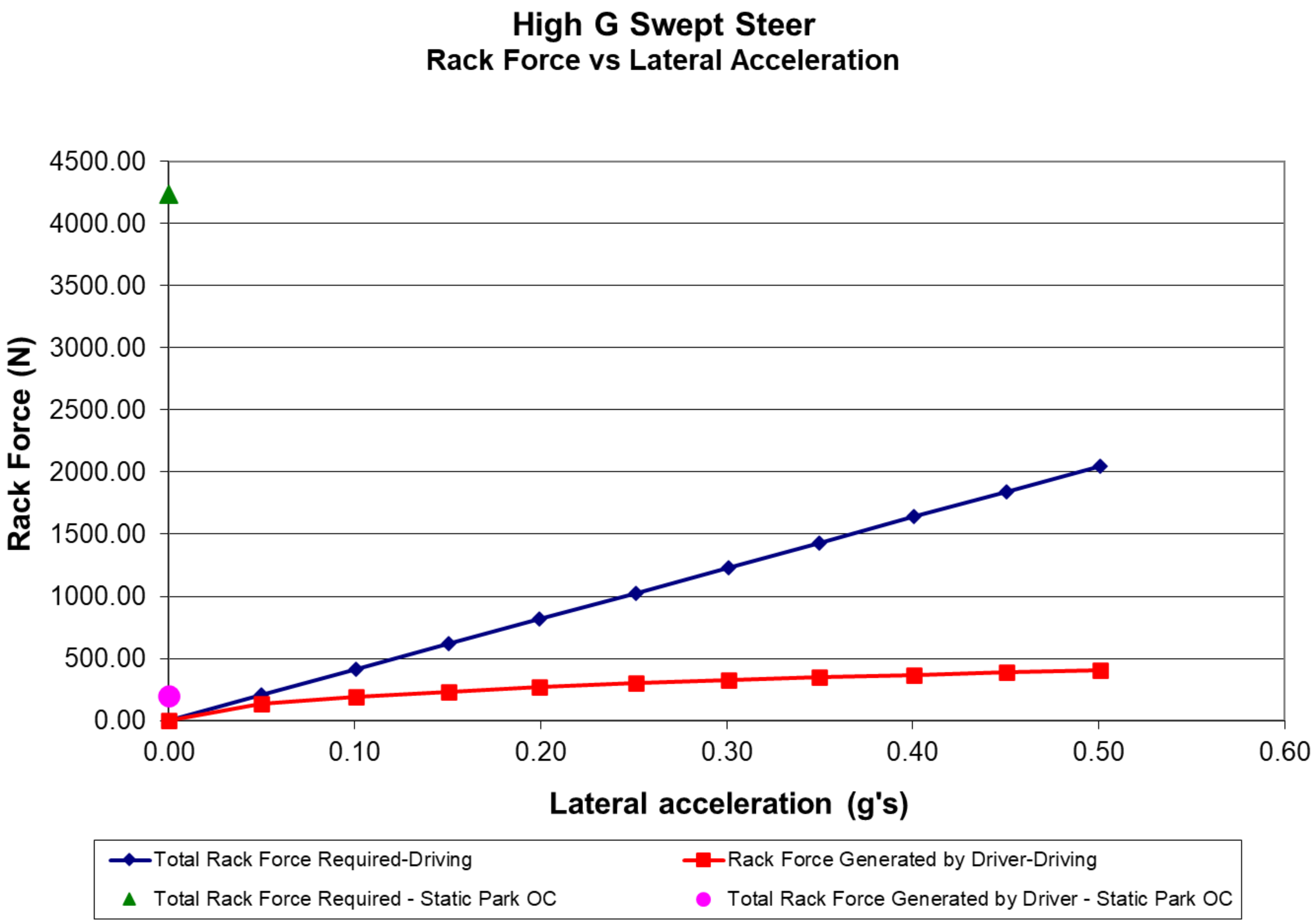
For a power steering system, the total output force generated by a rack and pinion steering gear is sum of the mechanical and assist force,

$$F_{r,p} = F_{r,m} + F_A$$

Where:

$F_{r,p}$ = Total rack force
$F_{r,m}$ = Mechanical rack force provided by the driver
F_A = Rack force provided by the power steering system

For next time…



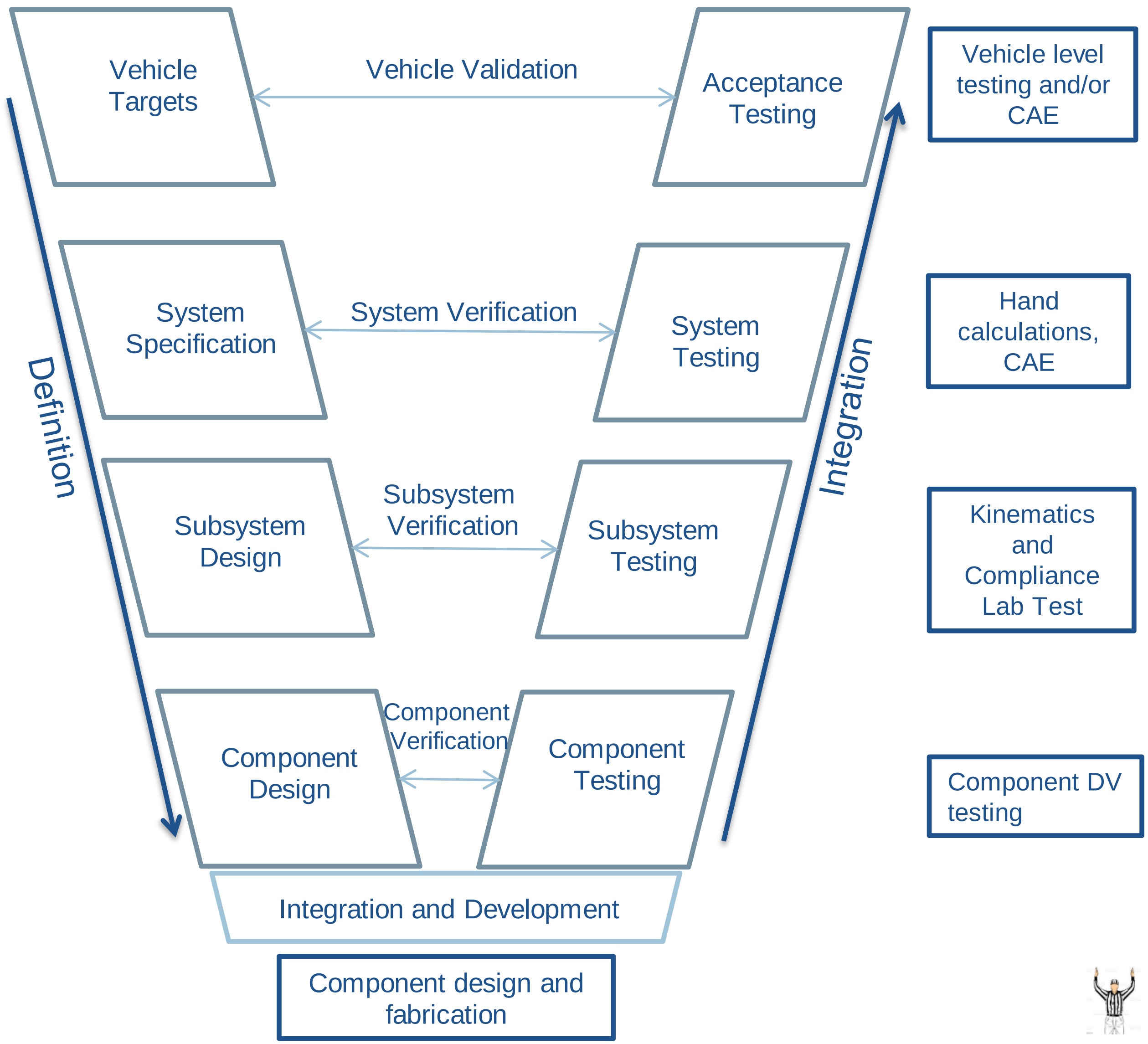
Systems Engineering V for Vehicle Dynamics – Steering Feel

Vehicle-level Targets:
- Cornering SWT @ 0.3g (Nm)
- Cornering SWT/Ay gradient (Nm/g)
- Max static/rolling parking effort (Nm)
- SWT to track on a crowned road (Nm)
- SWT due to torque steer (Nm)
- SWT with uneven brake torque (Nm)
- SWT nibble/wheel fight (Nm)
- …

Chassis System Targets:
- KP moment due to Fx, Fy and Fz (Nm)
- KP moment due to Mx, My, Mz (Nm)
- Front weight and weight distribution (N, %)
- Cg height (m)
- Front roll center height (m)
- …

Suspension & Steering Subsystem Targets:
- KP inclination angle (deg)
- Caster angle (deg)
- Kingpin offset (mm)
- Mechanical trail (m)
- Scrub radius (m)
- Steer arm length, gradient (m, m/deg rwa)
- Upper/lower steering friction at SWA (Nm)
- …

Component Specifications
- Suspension hardpoints (xyz)
- Lwr steering hardpoints (xyz)
- Strg gear ratio (mm/rev)
- Tire force and moments
- Power steering assist tuning
- Steering column friction
- I/shaft friction
- Steering gear friction
- Ball joint friction
- …



Summary

- The kingpin axis is the 3-dimensional elasto-kinematic axis about which the wheel rotates

- The Scrub Radius, Kingpin Offset and Kingpin Inclination Angle are important suspension parameters defined by the rear view location of the kingpin axis relative to the wheel center and contact patch

- The Caster (mechanical) Trail, Hub Lead and Caster Angle are important suspension parameters defined by the side view location of the kingpin axis relative to the wheel center and contact patch

Summary

- The total kingpin moment can be calculated by superimposing individual moments due to
  - Lateral, longitudinal and vertical forces at the contact patch
  - Drive torque at the wheel center
  - Tire aligning torque

- The kingpin moment is converted to a force by way of the steer arm.

- The length of the steer arm is determined by drawing a line normal to the kingpin axis and thru the outer tie rod ball joint

Summary

- A steering system is referred to as **front steer** when the outer tie rod ball joint is located **forward** of the wheel center

- A steering system is referred to as **rear steer** when the outer tie rod ball joint is located **aft** of the wheel center

- The force at the steer arm is a vector and has X, Y and Z components

- The lateral component of the steer arm force is counteracted by forces generated by the steering system via driver and power assist system